University of Alberta

Computer Graphing in High School Mathematics Classrooms: Responses of Teachers and Students

by

Marian Jean Oberg ©

A thesis submitted to the Faculty of Graduate Studies and Research in partial fulfillment of the

requirements for the degree of Master of Education

Department of Secondary Education

Edmonton, Alberta

Spring 1996

National Library of Canada
Acquisitions and Bibliographic Services Branch
395 Wellington Street
Ottawa, Ontario
K1A 0N4

Bibliothèque nationale du Canada
Direction des acquisitions et des services bibliographiques
395, rue Wellington
Ottawa (Ontario)
K1A 0N4

Your file   Votre référence

Our file   Notre référence

The author has granted an irrevocable non-exclusive licence allowing the National Library of Canada to reproduce, loan, distribute or sell copies of his/her thesis by any means and in any form or format, making this thesis available to interested persons.

L'auteur a accordé une licence irrévocable et non exclusive permettant à la Bibliothèque nationale du Canada de reproduire, prêter, distribuer ou vendre des copies de sa thèse de quelque manière et sous quelque forme que ce soit pour mettre des exemplaires de cette thèse à la disposition des personnes intéressées.

The author retains ownership of the copyright in his/her thesis. Neither the thesis nor substantial extracts from it may be printed or otherwise reproduced without his/her permission.

L'auteur conserve la propriété du droit d'auteur qui protège sa thèse. Ni la thèse ni des extraits substantiels de celle-ci ne doivent être imprimés ou autrement reproduits sans son autorisation.

ISBN 0-612-10789-2

Canada

University of Alberta

Library Release Form

Name of Author: Marian Jean Oberg

Title of Thesis: Computer Graphing in High School Mathematics Classrooms:
Responses of Teachers and Students

Degree: Master of Education

Year this Degree Granted: 1996

Permission is hereby granted to the University of Alberta Library to reproduce single copies of this thesis and to lend or sell such copies for private, scholarly, or scientific research purposes only.

The author reserves all other publication and other rights in association with the copyright in the thesis, and except as hereinbefore provided, neither the thesis nor any substantial portion thereof may be printed or otherwise reproduced in any material form whatever without the author's prior written permission.

Marian J. Oberg

~~5161~~ - 10 Avenue
Edson, Alberta
T7E 1K1

January 3, 1996
Date

University of Alberta

Faculty of Graduate Studies and Research

The undersigned certify that they have read and recommended to the Faculty of Graduate Studies and Research for acceptance, a thesis entitled Computer Graphing in High School Mathematics Classrooms: Responses of Teachers and Students submitted by Marian Jean Oberg in partial fulfillment of the requirements for the degree of Master of Education.

Dr. S. E. Sigurdson

Dr. T. E. Kieren

Dr. D. Sawada

December 15, 1995
Date

Abstract

The main purpose of this study was to develop and test classroom-ready materials in which Zap-A-Graph™ was used by students as a tool for graphing and transforming functions. Students and four teachers from two Grade 10 classes and three Grade 11 classes participated. The researcher's experiences using revised materials are included as a case study.

Students found using the software fast, accurate and easy. Teachers liked the variety introduced by computer graphing. Both noted that good explanations and examples were needed, along with class discussion and writing activities to clarify conclusions. Students needed to "learn how to learn" using the computer as an investigative tool, perhaps because their computer experience was primarily using application programs such as word processors or games.

As part of the study, teachers were asked about reluctance to try new ideas in teaching mathematics. They mentioned "risk" and "time" (both to prepare and to present) as concerns. They thought of good professional development in terms of dynamic speakers or well-structured workshops rather than personal or collegial activities.

Acknowledgements

I would like to recognize the contribution of the Board of the
Yellowhead School Division No. 12
who granted me a funded leave to pursue graduate studies.

Thank you also to those teachers and students who participated in the study.

I would like to thank Dr. A. Olson and Dr. S. Sigurdson for acting as my advisors.

I would like to dedicate this thesis to my family:
my husband Roger who has always supported my educational endeavours,
my children, Kirsten and Catherine who encouraged me,
and my parents, James and Dorothy Dey, who first taught me to value education.

TABLE OF CONTENTS

I. THE PROBLEM 1

- Introduction 1
- The Purpose of the Study 2
- Significance of the Study 4
- Delimitations of the Study 4
- Limitations of the Study 5

II. REVIEW OF THE LITERATURE 6

- Using Technology in High School Mathematics 6
- Using Computers in Studying Plane Geometry 6
- Using Computers in Studying Calculus 7
- Using Computers to Study Algebra 7
- Using Computers to Transform Functions and Relations 8
- Gender Issues 10
- Concerns About the Use of Computer Technology 10
- The Classroom Teacher 11

III. DESIGN OF THE STUDY 15

- Overview 15
- Creating the Materials 16
- Unit Design 16
  - Underlying Principles 16
    - Writing By Students As a Learning Activity 17
    - Questioning to Promote Student Learning 17
    - Student Exploration 17
    - Self Directed Activities 18
- Design of the Materials 18
  - Unit 1: "Learning About Lines" 18
  - Unit 2: "The Significant Seven" 20
- Implementation 21
  - Evaluation of Student Progress 23
  - Assessment of the Study 23
- Case Study 24

IV. INTERPRETING RESULTS 25

- Teacher and Class Biographies 25
- Students Response to Using the Computer 26
  - Student Background of Computer Use 27
  - Working With Partners 28
  - Exploring Beyond the Assignments 29
  - Interpreting the Student View of "Learning About Lines" 30
- Interpreting the Student View of "The Significant Seven" 31
- The Teacher View of Using the Materials and Methods of the Study 34
  - The Best and Worst of the Teaching Materials 34
  - Changing Teaching Practice - Why are teachers reluctant? 36
  - Changing Teaching Practice - What is the best form of professional development? 38

V. CASE STUDY 40
  Teacher and School Biography 40
  Updating and Materials 40
  Using the Materials 41
  Implementation 42
  Results 42
  Recommendations 44
  Reflections on the Case Study 46
  Changing Teaching Practice 47
    The Mathematics 10 Experience 47
  Professional Development 47

VI. SUMMARY AND DISCUSSION 49
  Restating the Problem 49
  The Materials and Their Presentation 49
    Presentation 49
    Questions 50
    Discussion 50
    Working In Pairs 50
    Variety in Instructional Practice 51
  Changing Teacher Practice 51
  How Teachers See Professional Development 51
    What Are Teachers Telling Us? 52
  Potential for Future Research and Development 52
  Summary 53

BIBLIOGRAPHY 54

APPENDIX 58
  Learning About Lines 59
  The Significant Seven 74
  Evaluation Form for Students 93
  Focus Questions for Teachers 95
  Curriculum Guide -- Coordinate Geometry and Graphing 96
  Curriculum Guide -- Functions and Relations 97

I. THE PROBLEM

Introduction

Constructivists suggest that students create their own knowledge from their own experiences. Younger students can use physical manipulatives to explore mathematical ideas and to construct their own schema. Physical modelling is more difficult to structure when students at secondary levels explore increasingly abstract ideas. Technology, specifically graphing calculators and computer graphing programs, allows the creation of the graphs of functions with an increased level of speed, and accuracy than is possible with paper and pencil techniques. Multiple representations of graphs, equations, and tables of values are available at the press of a button. Is it possible to use this technology to help students construct their own learning about functions?

Despite the potential of using graphing software as a tool, and the encouragement of leaders in the field of mathematics education, teachers have not always embraced the use of computers and graphing calculators with enthusiasm. There are many suggestions as to why: lack of knowledge about computers and how to use them in instruction (Schofeld & Verban, 1988, Fullan, 1992; Tall, 1987), logistics problems in setting up computer sessions, need for better hardware or software (NCTM Standards, 1989; Demana & Waits, 1990), concern about student evaluation (Tall, 1987; Schofeld & Verban, 1988), lack of knowledge of the potential of technology to improve instruction (Schofeld & Verban, 1988), or the belief that technology has little to offer mathematics instruction. How can this reluctance be overcome?

Teachers today are overwhelmed with new ideas, some of which are not compatible, or which work in opposition to each other. Although the current view of assessment in education is one in which a variety of evidence is used in preparing a profile of student performance, often the reality is still a strong emphasis on examination results by those charged with evaluating pupils or programs. Declining dollars in education have caused increased class sizes and teaching loads, yet teachers are encouraged to teach in ways which require more individualization. It is not surprising that teachers are sometimes unwilling to take the risk of trying something new, especially if that "something new" involves a heavy expenditure of teacher time and energy, or a significant amount of classroom time.

The time and energy required for the creative process to go from an idea to a usable classroom activity is often underestimated. When this gap is bridged by the creation of classroom-ready materials, teachers are more willing to try something innovative. Such materials represent a "first draft" for the teacher, in which a general idea is exemplified through activities or exercises designed for students. As teachers become more familiar with the innovation, they often modify the materials to suit their own teaching style.

The Purpose of the Study

This study follows the development of an idea, that of using computer graphing software in high school mathematics instruction, from theory to practice. Teaching materials which use computer graphing software as a tool in achieving the objectives outlined in Alberta curricula for the study of relations and functions were created by the researcher. Following the creation of these materials, the researcher , with the cooperation of four teachers, used them in high school classrooms. During the classroom sessions, the teachers often acted as both student and teacher as they learned about the special features of the computer graphing program Zap-A-Graph™ along with their students. In order to determine if these materials had potential as a teaching and learning device, both students and teachers were questioned regarding their use. A secondary objective of this study was to investigate how teachers accepted the materials created, and how they viewed these and other efforts to change their practice, both by themselves and by others. Finally, the researcher's own experience in using the materials is included in order to ground the experience as part of ongoing practice.

Four questions emerge in an effort to satisfy the purpose stated above. Included with each question is a description of the method used in attempting to answer it.

1. Is it possible to create teaching materials to use with commercial computer graphing software which will allow students, in an independent manner (with the teacher available for consultation), to explore the behavior of those functions they are required to investigate in the Alberta Senior High Mathematics Curriculum? Can this

be accomplished with available and economical computer software which will produce and transform graphs of commonly used functions?

To address these questions, the researcher set about to design and try out two units of classroom ready instructional materials for use with Mathematics 10 and 20 students during their study of the regular curriculum. Both use the computer program Zap-A-Graph™, a program for Macintosh computers, which is licensed to Alberta Education for use in Alberta schools at no additional charge to the schools.

Zap-A-Graph™ was developed by Michele Pitre as a teaching tool. As well as the obvious advantage of being readily and cheaply available, Zap-A-Graph™ carries out the usual graphing tasks required of high school mathematics students and allows the operator to transform the graphs of a wide variety of functions. This allows the student to work with the more concrete expression of the function, the graph, and to see the result of the transformation on the more abstract form, the equation.

The print materials which were developed in this study contain both instructions on the manipulation of Zap-A-Graph, and activities designed to promote the investigation of the functions and transformations required by the Alberta curriculum.

2. How do students rate the various aspects of this method of instruction as a way of learning, when it is compared to the usual classroom experience?

After the materials were developed, they were used with five classes, two of Mathematics 10 and three of Mathematics 20. Students in the five classes were questioned about their use of the materials. In a follow-up case study with the researcher's own classes, the materials were used. This experience has been included in the Case Study in Chapter V.

3. How do teachers judge the various aspects of using computers as an instructional tool with these support materials? Is this a better method of instruction than their usual presentation?

Four teachers were interviewed regarding their reaction to the materials and their use in the classroom.

4. Considering teacher reaction to the materials and their implementation, how do teachers respond to attempts to change their practice? Teachers were asked about

the kind of professional development necessary to use these and other new ideas and materials effectively.

5. How does the case study compare to phase one of the study where the researcher was not the regular classroom teacher? What changes occur when the researcher takes the materials and methods into her own classroom? Is the everyday reality different from the research situation? A case study, incorporating the researcher's own experience in revising and using these materials in her own classes of Mathematics 10, 20, and 33 is presented in Chapter V, where the questions above are reconsidered.

Significance of the Study

Despite the enthusiasm for the use of computers in secondary mathematics education by some leading educators, there has been surprisingly little research carried out on its use in the high school setting (Fey, 1989). In addition, few materials exist which are ready to use in the classroom, although the literature contains many suggestions as to how computers might be used. The production of classroom ready materials should facilitate the use of computers in high school classrooms by providing a place for teachers to investigate changing practice.

There are many ideas available to teachers who would seek to improve the way they teach but there is often a wide spread between idea and practice. A teacher faced with an already overcrowded professional life must seriously weigh the cost of embracing new practice (Fullan, 1991). How many hours of preparation will be required, first to become comfortable with the concept, to envision an approach which will be viable, then to prepare the materials which will bring the idea to the students in the classroom? This commitment is undertaken with the understanding that any new approach may be flawed and prove to be a waste of time.

This study is about bridging the gap between theory and practice. It is intended to give teachers the opportunity to try out an idea with a minimum of risk. Given the demands of teaching, this collaboration between researcher and teacher is essential if research is to remain relevant to the classroom (Kabaroff, 1992).

Delimitations of the Study

1. Materials were delimited to fit with topics in the current Alberta curriculum for Mathematics 10, 13, 20, and 23 and with approved textbooks.

2. This study examines only one application of one computer graphing program in mathematics education. The conclusions drawn here can not be considered true of computer applications to mathematics education in general.

3. The units designed represent the researcher's choice, interpretation, and use of the software. They may not represent the best possible examples of the use of computer graphing in teaching these topics.

Limitations of the Study

1. The teachers involved in the study have volunteered to participate so they have already demonstrated an open attitude to the use of computers in their classrooms and to the possibility of embracing new ideas.

2. It is important to this researcher that the project be close to each teacher's "comfort zone" (and that of their classes). Thus, some decisions were made for reasons other than research goals. (For example, all teachers were concerned that time spent on these activities should not detract from other objectives in the course.)

3. Teacher perceptions were determined by informal interview and student perceptions were determined by written comment in response to researcher prompts.

4. The program was offered once with each teacher (other than the researcher's experience as noted in the Case Study, Chapter V.)

5. The researcher carried out most of the teaching when working in the classes.

II. REVIEW OF THE LITERATURE

The NCTM, in its document Curriculum and Evaluation Standards for School Mathematics, suggests that computers and calculators should be available to all students as tools with which to explore the changing discipline of mathematics. Throughout the document, reference is made to ways in which student investigations can use technology "...to investigate, conjecture, and verify their findings" (Standards, page 128). The power to explore mathematics independently is now available to students; the Standards suggest that the new mathematics curricula should encourage and guide this exploration.

Using Technology in High School Mathematics

There has been much written about the use of computer graphics to enhance mathematics instruction since the microcomputer with viewing screen became available at reasonable cost and many claims have been made regarding potential benefits. The students who learn to use this technology will become more self-directed and more self-monitoring (NCTM Report, 1985). Their thinking will become more versatile (Blackett, 1989). They will become better problem solvers (Waits and Demana, 1988). The potential for providing visual models of algebraic functions will help those (more often female) who ordinarily have problems with visual-spatial tasks (Ruthven, 1990). Most authors assume strong motivational benefits as students become engaged in problem solving. Student attitudes to computers and instruction on computers seem to improve although not necessarily attitudes to the subject matter (Bennett, 1991). Studies of a number of possible topics which might involve computer instruction have been carried out, including geometry, algebra, calculus, and the study of functions.

Using Computers in Studying Plane Geometry

One example is the computer language Logo which allows children, from preschool to high school, to investigate mathematics, particularly geometry. Important research on using Logo has been carried out by Hoyles and Sutherland in Great Britain. The work in their project raises interesting considerations for anyone working with computers, such as questions of gender differences, effectiveness of student planning strategies, group dynamics, and evaluation of results, to mention a few. (Hoyles and Sutherland, 1989)

Geometric construction and exploration tools are another type of graphics software which allow students to carry out investigations on geometric shapes. With these tools, students use the computer to carry out constructions and measurements in making and proving their own conjectures about figures in the manner of mathematicians (Fey, 1989). To date, much of this exploration follows a traditional approach, with the computer making the constructions more quickly, easily, and accurately than the student might.

Using Computers in Studying Calculus

Exploring calculus through computer graphics allows the development of the idea of derivative before the formal discussion of limit. In his work in this area, David Tall (1985) suggests that initially, differentiable functions be examined by looking at them "close up" so that the sections of the graph appear as straight lines for which the slope can be easily determined. Later, the computer calculates and plots the slope of a moving chord drawn between two points on the curve, thus leading the student to discover the derivative of a function from its graph. The work of Blackett (1989) continues this exploration. Another study, in which computers were used to study calculus, found that students who used a computer to perform algorithms and graph functions were able to answer conceptually oriented questions appropriately and with confidence (Heid, 1988).

Using Computers to Study Algebra

Several research projects have been undertaken to demonstrate the potential of these programs. In the early 1980's, an article in the *Mathematics Teacher* (Kennedy, 1981) included a graphing program which would draw polynomial graphs, to be used in support of the regular investigation of polynomials by traditional methods. By the mid 1980's, several researchers were involved with the use of commercial software to facilitate student exploration of concepts.

A technique used to investigate the polynomial functions usually studied in high school is that of building polynomial graphs from monomial parts (Dugdale, Wagner, and Kibbey, 1992). Using graphing software, individual monomial terms are plotted and compared with the result when they are combined by addition. With experience, students learn to create polynomials which have predetermined characteristics. Interesting situations can be explored using the products and quotients of polynomials as well.

As technology became more available in schools, examples promoting the use of graphing computers or calculators began appearing in professional literature and textbooks. Creative uses of the technology are seen frequently, although there may be little mention of related research. Equations can be solved by graphing each side individually and determining the intersection points of these two representations (Waits and Demana, 1990) and the algebraic solution of systems of inequalities becomes more understandable when graphed (Hector, 1992). Identities can be verified by graphing both sides and demonstrating that the two graphs are composed of exactly the same points (Hector, 1992). Zeros of graphs, previously determined by formula or factoring, can be found by investigating the x-intercepts of the graph of a relation (Hector, 1992). Similarly, maximum and minimum values can be explored long before formulas or derivatives are mentioned (Waits and Demana, 1990).

Using Computers to Transform Functions and Relations

In the senior high school curriculum, particularly in Alberta (Alberta Education, 1989, 1990), some of the more useful tools are the programs which graph relations and functions for use with algebra and coordinate geometry units.

The speed and accuracy of computer graphs can help students generalize about the behavior of relations (Waits and Demana, 1988) and allow them to take an active role in mathematics. In addition, the understanding of the mathematics involved is deeper (Waits and Demana, 1988, Fey, 1989) Research with students who were introduced to linear equations using graphing software produced better results on related tasks than did control groups taught by an experienced teacher in a more traditional manner (Blackett, 1989).

Transformations of standard functions provide an interesting way for students to explore graphs (Bloom, Comber, and Cross, 1986). In this approach, the student begins with the simplest form of the function. From there, transformed images can be created, and their algebraic representations studied. With the right software, students may also modify a standard function to make it fit one suggested by the computer program.

In his research with graphic calculators, Kenneth Ruthven reports superior performance by a group using the calculators in both recognition and refinement of graphs commonly studied at the high school level (Ruthven, 1990). The relationship between the algebraic representation of functions, graphs, and tables of values can be enhanced by the use of multirepresentational software to study contextual problems (Rizzuti, 1991; Demana and Waits, 1990).

Elfriede Wenzelburger Guttenberger (1991, 1992, 1993) writes of a series of studies in which students used the program "Cactusplot" to study transformations of trigonometric functions. In one study, she worked with 31 students, 8 of whom were chosen for the computer group (Wenzelburger, 1992). The computer group used a study guide in which the students were guided in their exploration of the parameters a, b, and c in the equations **y = asinbx** and **y = acosbx** as well as having the investigator available for consultation. Students in the computer group did significantly better on the posttest than the classroom group who had teacher instruction only. Three months after the posttest, a surprise retention test was given; the computer group achieved significantly better than the classroom group, especially on questions which required the change between graphic and symbolic form. Within the computer group, girls achieved better on the posttest than boys but this finding did not extend to the retention test.

In a later study, Wenzelburger (1993) reports a similar study, this time with 25 students, 14 in the computer group. Students studied the trigonometric functions **y = asinbx + d** and **y = acosbx + d** according to a three-stage learning model: free exploration, analysis, and comparison. Again, the computer students did somewhat better on the posttest and much better on the retention test given four months later. The stronger performance was most evident on questions requiring the interpretation of graphs, and on some, but not all, of the questions requiring interpretation of equations. There was no indication of separation of results by gender in this study.

In comparing the two studies, the only objective in which the computer group excels in both is the task of identifying or writing the equation from the graph. Otherwise, although the computer group has stronger performances on certain objectives in each study, those objectives do not appear to be the same in both. Certainly the fact that students are able to view many more examples would give credence to that one objective in which the computer group is strong in both studies. Perhaps further studies will be more definitive in exactly what benefits students are receiving from their computer experience.

Gender Issues

Although there is considerable dispute over the issue of gender in mathematics education in general, there is some agreement that there are students for whom spatial visualization is difficult and that more of these are female (Battista, 1990; Ruthven, 1990). These authors suggest that using graphing calculators or computers can help to compensate for this problem; however, research on attitudes toward computers (Collis, 1987) suggests that perhaps negative attitudes toward computers (held more often by females) may transfer to the mathematics class. We may end up creating more problems with our attempts to help.

Concerns About the Use of Computer Technology

In a 1989 survey of developments in the field of technology in mathematics education, James T. Fey comments that "...there is very little solid research evidence validating the nearly boundless optimism of technophiles in our field" (Fey, 1989, page 237). Many explorations in the use of technology in the classroom contribute to the general enthusiasm surrounding its use; however, many teachers have yet to be convinced of its value.

David Tall advises: "If we are not careful, computer education could become...flashy technology which speedily attains a result without demonstrating the richness of meaning that lies at the root of it." (Tall, 1983, page 40). The necessity of being aware of students' previous experiences and abilities before using particular methods is suggested by Harold Frederick (1989). The need for providing accompanying worksheets for projects and good teacher guidance is mentioned by Dreyfus and Halevi (1990). Students will need to develop new skills in exploration and inquiry which are not currently emphasized if they are to be able to make use of technology in pursuit of their own learning. The computer-based graphics tool itself is not sufficient to improve the student's grasp of mathematical concepts; in fact the technology may often be misleading when used superficially. (Colgan, 1993).

The necessity for students to reflect on and record the results of their explorations using the computer is not always addressed in the literature. Few authors discuss the mundane subject of follow-up exercises and suitable homework. Are we to assume that everything can be learned on the computer? Because computers are seen as agents of change, it should not be assumed that we will either continue exactly as we have in the past, with a few hours of computer exploration replacing "teacher talk", or that we will turn everything over to exploration on the computer. What about students who do not learn well in the computer environment? Students have to function at a higher cognitive level more often when constructing concepts for themselves than was required in a more traditional setting. How can they be supported to become more mature learners? (Bassarear, 1987). What about students who are frequently absent and miss exploration activities? How can they be involved in negotiating the meaning of activities they have missed? What about assessment? Both teachers and their students are concerned with possible changes when an innovation is introduced (Bassarear, 1987).

Suggestions have been made that students learn better and faster when the computer is used to supplement regular instruction (Bennett, 1991). Summary statements of this sort may be more dangerous than helpful, as they combine results of drill and practice, tutoring programs, programming experiments, along with the use of the computer as an exploratory and problem solving tool. We need more research about exactly how students who are presently encountering learning problems in mathematics can benefit from the use of software in understanding concepts and solving problems. Is it really true that students who were previously disinterested get involved and motivated when using function graphing software to explore?

In fact, students' attitudes may not be supportive of teaching methods involving constructing their own learning if these methods are not familiar to them. Not only are these methods suspect because of their novelty, but they may resent the extra "work" they have to do in teaching themselves. After all, isn't this the teacher's job (Bassarear, 1987)? Wenzelburger concludes, "...information technology can be useful in mediating mathematics in the classroom, but ... much thought must go into the way it can be used most effectively,"(Wenzelburger, 1992).

The Classroom Teacher

For those of us who feel the computer has something to offer high school mathematics students, the question becomes, "How can we use the computer effectively in our instruction?" Repeatedly in articles describing the use of technology, teachers are encouraged to get involved. "Teachers must start now to implement the many technologies currently available and prepare for the explosion of technology to come in this decade." (Waits and Demana, 1990, page 27). "Teachers must become effective catalysts for student-directed learning." (NCTM Report, 1985, page 247). Exactly how teachers can become involved and why they should want to is not always evident.

Most authors suggest that the role of the teacher will change with the introduction of the computer in the classroom. The teacher will become a guide, and a fellow learner who explores new concepts in partnership with students, encourages discussion, and facilitates group process (Fey, 1989; Waits and Demana, 1988; NCTM, 1985). The teacher will be a technician, sorting out problems of hardware and software, as well as mathematical content (Heid, Sheets, and Matras, 1990). The teacher's role becomes increasingly important, because it is the teacher who must ask the right questions at appropriate moments to help students define the direction of their explorations. If the students are to construct their own learning so that it fits the "taken-as-shared" practices and symbolism of the larger mathematics community, it is necessary for the teacher to be actively involved in negotiating meaning with the students (Cobb, Yackel & Wood, 1992).

In his book, The New Meaning of Educational Change (1991), Michael Fullan talks about change and why we need to understand what it means for teachers to change their practice. He states, "Change is a highly personal experience--each and every one of the teachers who will be affected by change must have the opportunity to work through this experience in a way in which the rewards at least equal the cost," (p. 127). Cost here refers to "...time, energy, and threat to sense of adequacy,..."(p. 129) and rewards refer to "...a sense of mastery, excitement, and accomplishment,..."(p. 129). In addition, teachers need to be sure that a potential change will benefit students, that procedures are clearly explained and that the change can be carried out with resources available .

Fullan speaks of three aspects of change: (1) new or revised materials, (2) new teaching approaches, and (3) altered beliefs. He makes the point that "...change has to *occur in practice* along the three dimensions in order for it to have a chance of affecting the outcome,"(p. 35-36). Of these, the third (change in beliefs) is the most difficult to facilitate, and may require some experience with the first two, before teachers are in a position to address it effectively (p. 42).

Teachers tend to be conservative, and there are forces which act to keep them fixed in their practice (Fullan, 1991). We all have our "comfort zone" (Hauk and Quinn, 1992) and even when we embrace the philosophy of increased use of graphing technology, changing our classroom practice is a challenge. When we are pulled from our comfort zone by others, the results can be disastrous. Understanding the culture of the group which is to undergo change and recognizing the role of the constituent subgroups is important to effecting change from outside the group (Sarason, 1971). Because it is so difficult to mandate change, recent literature advises that teachers should be "empowered" to make changes from within the system. In order to be empowered, teachers need increased knowledge of their subject, knowledge of the "system" and collegial relationships with others both inside and outside the education community (Lichtenstein, McLaughlin, and Knudsen, 1992).

For teachers to become involved in using computers in instruction, a great deal of support of various kinds is required (Fullan, Miles & Anderson, 1988; Schofield & Verban, 1988; Fullan, 1991). Students often resist a new way of learning at the time when teachers are most vulnerable because they are using a new way of teaching (Bassarear, 1987). Teachers must become comfortable using the hardware and software themselves before using it with students. It is also necessary to provide ideas and examples of how students can use and benefit from time spent with the computers. Traditional materials may not be useful for computer investigations. Contact with colleagues is important for emotional support as well as sharing information. Interaction among teachers is strongly related to the success of any change in practice (Fullan, 1991).

Using computers in instruction leads to a different type of classroom interaction and a different form of evaluation (Fullan, Miles & Anderson, 1988) which may prove an obstacle for those who wish to increase computer use. The NCTM Professional Teaching Standards (1991) considers computers and other technologies as a means of enhancing discourse about mathematics (described as "...the ways of representing, thinking, talking, agreeing and disagreeing..."(p. 34)). In Algebra in a Technological

World (1995) the teacher is seen as "...catalyst and facilitator of learning," (p. vi), rather than a "...dispenser of information,"(p.vi). The NCTM, in Curriculum and Evaluation Standards in School Mathematics (1989) advises that, if materials such as calculators and computers are used in instruction, they should also be used in assessment and that the assessment tasks should be appropriate for use with these devices. These references also note that learning when and how to choose and use technology is an important part of carrying out any mathematical task.

III. DESIGN OF THE STUDY

Overview

For new ideas to become part of teaching practice, it is necessary for these ideas to be translated into usable classroom techniques and lessons which teachers incorporate into their teaching practice. Therefore, there are five questions to be answered regarding the concept of using computer graphing in high school mathematics throughout this process:

1. Is it possible to create lesson materials for students in which they explore the functions specified in the high school mathematics curriculum in order to achieve the outcomes required? The outcomes to be addressed in the materials are that:
    a. "Students will be expected to design and carry out an investigation involving the use of scientific calculators or computers to determine the effects of the parameters **m** and **b** on the graphs of linear equations", (Alberta Education, 1989, p. 21). and
    b. "Students will be expected to demonstrate an understanding of how particular parameters can be used to effect translations, reflections or vertical stretching of the graph of any function" and "Students will be expected to predict the graphs of functions written in the form $\mathbf{y = cf(x - a) + b}$ given the graph of $\mathbf{y = f(x)}$, and verify using calculators or computers", (Alberta Education, 1990, p. 60).
2. How do students rate this method of instruction when compared with their regular classroom experience? What computer background and attitude do they bring to the class? How do they view the materials and their presentation?

    Students who used these materials were given a brief questionnaire to find out about their previous computer experiences and to allow them to react to the materials.
3. How do teachers rate using computers with the study materials provided? Teacher interview questions allowed them to react to the materials and discuss the potential for computer graphing as a support to teaching.
4. Keeping in mind the teacher reaction to the materials provided and the teaching methods used, how do teachers respond to attempts to change the manner in which they practice?

Teachers were asked about the kind of professional development necessary to effectively use these materials in their classrooms. They were also given the opportunity to talk about professional development in general, in an attempt to probe the question of how to get new ideas into practice.

Creating the Materials

"Educational change depends on what teachers do and think...," (Sarason, in Fullan, 1991, p. 117). The two units developed in this study are its cornerstone. They have been created in an attempt to change what teachers "do" in their classrooms. The materials were not intended as a finished product to be used indiscriminately, but rather as a place for teachers to start to change their practice in incorporating technology into instruction. Because the acceptance of the idea of using computer graphing in an exploratory manner depended on the success of the experience, their development required considerable attention.

Unit Design

To guide students in achieving the outcomes indicated at the beginning of this chapter, two units were written. The first, "Learning About Lines", was written to guide students in the exploration of the line, particularly to investigate the transformations of vertical stretch and vertical translation, which cause the slope and y-intercept of the line to change. The second, "The Significant Seven", investigates the transformations of vertical stretch and horizontal and vertical translation on seven commonly used functions: the line, the parabola, the cubic function, the exponential function, the absolute value function, the square root function and the rational (reciprocal) function.

The unit design required many decisions, including choice of activities, type and amount of explanation, questions to be asked, and sequencing and presentation. These decisions were guided by a number of underlying principles.

Underlying Principles

These units were based on several teaching principles including writing by students, student exploration, and the use of self-directed activities which are described in the sections which follow.

Writing By Students As a Learning Activity

Writing can be used by students as a means of consolidating the ideas gained from the activities. Writing "...requires students to reflect on, analyze, and synthesize the material being studied in a thoughtful and precise way," (Davison and Pearce, 1988). Whenever possible, the students are asked to write a sentence or two to summarize the results of an action carried out on a series of graphs. In the early stages of the unit, students are asked to fill in charts to summarize information before being asked to write so that they will have the information gathered before being required to write. An example of this occurs in the Mathematics 10 materials, "Learning About Lines". Students are asked to "Describe what happened as you changed the vertical stretch factor. Talk about the change in the position of the line, the change in the equation, and the change in Δx."

Spaces are left in the workbook to encourage students to write answers rather than simply to think about their results. The intent of the writing activity is to slow student progress and demand that ideas be translated into words, in order to promote both understanding and remembering.

Questioning to Promote Student Learning

A second principle is the attempt to include "good" questions about the activities. Some are included to promote reflection, for example, "What is the result of the vertical stretch of 2? (to the graph and to the equation)" which follows the investigation of $y = |x|$ when stretched. Others tie the ideas to past learnings,such as the review questions on slope and y-intercept which are included when students are learning to graph lines on Zap-A-Graph™ at the beginning of the Mathematics 20 materials, "The Significant Seven", where slope and y-intercept is reviewed. In some instances, students are asked questions to anticipate future study. For example, asking students to examine the change in y as x changes is useful not only in the study of the functions at hand, but also as a forerunner of the ideas in calculus.

Student Exploration

The strongest underlying principle is that students should be actively involved in their own learning whenever possible. Despite the influence of the constructivists on current mathematics curriculum, it is not as common for teachers at the high school level to use exploratory activities (such as the sort that Skemp structured for primary children (Skemp,1989)). This may result from a traditional method which tended toward the teaching of algorithms followed by practice, and relegated the exploratory methods to primary classrooms. Many of the understandings developed come by extending previously developed schema through a formal reasoning process often expedited by the teacher. Exploration such as using computer graphing can be valuable in allowing students to experience first-hand the development of their own understandings.

Self Directed Activities

Because this instruction will be delivered in a laboratory situation, the materials should be all-inclusive, much in the manner of computer assisted instruction (CAI) where all the objectives would be addressed within the workbook and the computer activities. The teacher's role is to clarify instructions, give individual help to students, and attempt to solve problems with the hardware and software. From a practical standpoint, this approach is mandated. Schools generally share computer facilities among many classes so that group instruction is difficult in the computer setting, due to additional noise, placement of computers, and general distractions in the room. In addition, with students exploring at their own pace, getting and maintaining their attention for all but the most necessary instructions is almost impossible and can interfere with the natural progress of their exploration.

Design of the Materials

Unit 1: "Learning About Lines"

The first unit of study, "Learning About Lines", was created to be used with the Coordinate Geometry unit in Mathematics 10 from the Alberta curriculum (Alberta Education, 1989). The lessons (see Appendix, pages 59 - 92) make maximum use of Zap-A-Graph™ (Pitre, 1991) to achieve the objectives of the curriculum. The software allows the student to transform the simplest form of the equation (in this case $\mathbf{y} = \mathbf{x}$) and to observe the results on the graph, the equation, and the table of values. By transforming the graph, as opposed to the equation, the student approaches the change as a physical activity.

Menus with the appropriate selections have been captured as part of the instructions so that students can proceed with a minimum of teacher instruction. Questions about the graph are included in the workbook to encourage student reflection on important understandings. In addition to learning about the behavior of lines, these lessons also introduce the software to the students, and, more importantly, introduce techniques of analysis useful in the future study of relations and functions.

Lesson A of "Learning About Lines" begins with graphing **$y = x$** using the software. Students are asked to call up the table of values for this function and are asked to look at the change in y as x changes by 1. There are also questions about quadrants and intercepts to reinforce these concepts which have been previously developed in the classroom introduction to coordinate geometry.

This provides the lead into Section B which uses the transformation called "stretch" by Zap-A-Graph™. The idea of stretching is new to the students and should be discussed by the teacher. Students need to recognize that Zap-A-Graph™stretches vertically, away from the horizontal axis, so that all points on the graph except those on the x-axis will move. The concept of slope is developed by looking at the change in y values as x values change by 1 when the "stretch" transformation is applied. This can be determined effectively from the table of values for each line. Students are asked questions which require them to examine the changes in the graph, in the equation, and in the table of values caused by stretching .

In Section C, students are asked to describe, in sentence form, what has happened as a result of vertical stretch. They are also given practice determining slope using the standard definition ratio of rise to run, and in finding pairs which belong to a particular line. Section D deals with the application of direct variation and related examples which places the theory about lines into context.

Part 2 of "Learning About Lines" begins by graphing $y = x$, followed by vertical translations of the line with the resulting change in y-intercept. Questions appear in the lesson as the new lines are drawn with the intent of focussing student thinking. Students are required to write about the change in the graph and the equation of the relation using the previous questions as a guide. Practice identifying slope and y-intercept from line equations of different forms and from graphs are included. Some of these exercises use Zap-A-Graph™, but others do not.

Part 3 of "Learning About Lines" includes activities to explore both horizontal and vertical translations of the same line, parallel and perpendicular lines, and concludes with exercises in which the students write their own equations of lines which have required characteristics. The materials often refer students to the textbook, Mathematics 10 (Kelly et al, 1989) .

The researcher quickly determined that not all the objectives for this unit could be presented effectively by means of computer exploration. To do so made the lessons artificial and unnecessarily convoluted. Some time had to be spent practicing algebraic manipulation and reinforcing previously learned skills. For example, students should be able to convert from equations of the form $Ax + By + C = 0$ to the form $y = mx + b$ having completed earlier work with polynomials. In practice, it is necessary to review and practice this skill if it is to be used as a means of determining slope from the equation. (Noting that the computer uses both forms of the equation might prove motivational, or might provide a means of checking the results of the algebra, but is no replacement for the paper and pencil drill.) Appropriate paper and pencil exercises were therefore included in the materials.

Unit 2: "The Significant Seven"

The second instructional unit was developed to meet the objectives of Concept 3 of the Mathematics 20 unit "Functions and Relation" (Alberta Education, 1990). Part of this unit requires that students become familiar with several common functions and examine the results of varying the parameters a, b, c in the form $\mathbf{y = cf(x-a) + b}$. The instructional materials are written to make use of the "Transform" commands in Zap-A-Graph™. The "normal" form of each of seven functions is transformed by vertical stretch, then by horizontal and vertical translation. (Horizontal stretch is not required of students in Math 20, although it might be explored as an introduction to the study of changes in period in trigonometric function.) The seven functions used are those of the linear, quadratic (parabola), cubic, absolute value, square root, exponential, and rational (reciprocal). The square root function is not specified separately in the curriculum guide, but does appear in the text used by these classes, Mathematics 11 (Kelly, 1990). It lends itself to studying translations because it has an easily recognizable reference point.

The first lessons involve graphing the seven functions, by hand from a table of values, and again using Zap-A-Graph™ menus (Appendix, pages 75 - 80). Not all of the functions are easy to call up; students are given instructions to produce the basic functions on the screen and are asked questions about them. The form chosen to draw the original equation is the form in which the translated equation appears, so students are required to follow the specific instructions given in order that future lessons produce usable results. Students have not studied these functions previously, with the exception of the line, and possibly the parabola.

In "The Significant Seven" the first lessons deal with vertical and horizontal translations. Students are advised to follow the movement of a reference point (for example, the vertex of a parabola) and the x- and y-intercepts to track basic functions. In each case, one of the seven functions is used as an example, and students are required to translate the basic function and fill in the chart to note the results. The equation appears on the screen (unless this feature is turned off), and a table of values is easily produced and modified. Students are asked to summarize the changes to the graph and to the equation as a result of the translation. In addition, students complete questions from their textbook, and a "Guess My Equation" exercise in which they are given reproductions of graphs or tables of values drawn on Zap-A-Graph™, and are asked to predict, then check, the equation which describes these.

Next, students transform functions using stretch (Appendix, page 85). They are required to answer questions regarding their observations, and to write summary sentences. Later, they carry out multiple transformations on "normal functions". They are also asked to recognize transformed functions from a graph already drawn or from a table of values.

Further lessons involve the use of "reflection", and a lesson which complements classroom work on the inverse of a relation. Zap-A-Graph™ allows reflection in the axes as well as in the line y = x, producing both the graph and its equation for each reflection.

Implementation

The teachers of two grade ten classes from a large urban school jurisdiction agreed to participate in the research. The idea of having the researcher take the responsibility for the teaching of the units was welcome; the teachers would work with the class and the researcher, learning about the software and this application of it. They would not have an additional burden placed on them, they would learn about something new which interested them, and yet they would be with the class to share the experience. In return, the researcher was present at all times to observe the way in which the students used the materials. This proved to be an excellent symbiotic relationship.

To conduct the research on the second unit, "The Significant Seven", two schools in a smaller jurisdiction, with a mix of rural and urban students, were approached. They were quite willing to participate and, because they knew the researcher personally, were easier to convince. Both teachers involved were quite happy with the format used on the first part of the study and looked forward to the break in their daily responsibilities.

Students worked in pairs for the most part. This was necessary because many of the labs did not have as many computers as there were students in the classes. Research suggests that there are definite benefits from this arrangement as students share ideas and discuss the meaning of what they see.

One of the first classes which used "Learning About Lines" was that of Teacher B. Her students had completed the study of the unit already, and spent only two class periods learning to use the computer program and reviewing the content of the unit on coordinate geometry. During the second period, a number of them were involved in another activity, so that they were not able to complete the evaluation.

Following an introduction to coordinate geometry by Teacher A in the classroom, students spent the following thirteen, 80 minute class periods working with the lessons prepared by the researcher or in related activities.

Not all of the days were spent in the computer lab because of prior bookings nor was all the time in the lab spent with the computer. The days in the classroom allowed for discussion, paper and pencil work, quizzes and a final test. Following the completion of the unit, students were asked to fill out a two-page questionnaire (see Appendix, pages 93 - 94) regarding their own computer use outside of class, and their response to this unit.

In the second phase of the study, Teacher C with his Mathematics 20 class, and Teacher D with two Mathematics 20 classes used "The Significant Seven" materials to study the transformation of functions of the form $\mathbf{y = cf(x - a) + b}$.

The completion of this unit required about eight days of a semestered Mathematics 20 program. Students worked from their textbooks in addition to completing the materials provided by the researcher, but most of these assignments were included to support the concepts developed by computer investigation. Most of the topics could be studied using the computer; very few required teacher demonstration.

Evaluation of Student Progress

Students were given unit tests based on those used by the researcher in non-computer classes. This was primarily to provide information to the classroom teacher for the usual school reports, and to assure both the teacher and the researcher that the learning objectives of the unit had been met.

There was no attempt made to use this as an instrument to formally evaluate the materials. The decision had been made to analyze the results of the research by means of qualitative methods so that no test was prepared to provide statistically reliable and valid results. The tests which were given sometimes included questions from topics which had been discussed by the teacher before the research began, and were provided for the convenience of the teachers involved.

Assessment of the Study

Throughout this study, the researcher worked with the class and the teachers on a daily basis. Much of the assessment was carried out by observing the reaction of the class to the materials and discussing their reactions with their teacher. By observing this closely, the researcher could adjust the materials to better suit the students as they were being created.

Students were asked to fill out a two-page questionnaire (see Appendix, pages 93 - 94) when they had finished using the materials. They were asked questions about their computer use prior to this study. They were then asked to rate the materials as "satisfactory" or "unsatisfactory", and given an opportunity to indicate the best and worst parts of using the computer to study coordinate geometry. One question asked if they explored the graphing program beyond what was required in the lesson. In conclusion, students were given the opportunity to play teacher and choose what they felt was the best way to teach the unit.

Teachers took part in an interview which began with a discussion of the "best" and "worst" parts of the materials. They also discussed the changing nature of mathematics, and whether using technology would mean teaching in a different way.

Before teachers can change practice, it is necessary for them to become aware of what researchers and colleagues are exploring and to become sufficiently excited about these new ideas to want to take the risk of trying them out. As part of the interview, teachers were asked questions regarding which professional development methods were most helpful in motivating them try out new practices (such as the ones in this study).

Case Study

The researcher used these materials in three of her own Mathematics 20 classes and in one Mathematics 33 class. This allowed her to observe the results of changes made in the method of presenting the materials. When students used the materials in the regular classroom setting and not as part of a research project, they were more likely to show their true feelings. These responses could be interpreted in the light of their reactions to other topics in the course. In the case study, the same teacher questions were addressed as in the original research, with particular attention paid to changes which were made to address weaknesses in the original materials. The case study is discussed in Chapter V.

IV. INTERPRETING RESULTS

The teachers who were identified to participate in the study satisfied three conditions. First, they were willing to allow the researcher to work with their classes to experiment with a new approach to instruction. (Considering that they are accountable for the instruction taking place with the students in the classes assigned to them, this does require a certain level of trust.) Secondly, a Macintosh computer lab was required for several days, as Zap-A-Graph™is available for Macintosh only. Finally, the students had to be at an appropriate point in their mathematics curriculum at the time when the researcher was prepared to carry out the research.

Each of the teachers and their respective classes represented a unique situation. A description of each situation follows in the next section.

Teacher and Class Biographies

Teacher A was an experienced mathematics teacher who had a mathematics major when he graduated from the Faculty of Education 26 years before. After a brief stint in junior high school, he proceeded to the senior high school where he had been teaching for 24 years at the time of this study. During those years, he had taught most of the mathematics courses in high school. His approach appeared to be quite traditional.

Teacher A's class of 22 students was one in a school located in the downtown area of a large urban centre, a neighbourhood which was home to a number of new Canadian families. Most of the students in the class were reasonably proficient in English, but for some it was not their first language. Their computer lab was part of the library, but in a self-contained area. It was necessary for some students to work with partners because there were not enough computers for everyone.

Teacher B also taught in large urban school, but in a middle class neighbourhood. She had begun her career in elementary school, then worked with special education classes. For the 14 years before this study, she had been teaching mathematics in high school, and had taught most high school mathematics courses. She mentioned using cooperative learning as a teaching model for some lessons, and seemed to be actively seeking new activities which would benefit her students. Because her Mathematics 10 class had already completed the unit on co-ordinate geometry by the time she was approached, she agreed to a two-class experiment where she and the students could try out the software and the materials. This class was an advanced placement class so found little difficulty in the content of the two lessons chosen for them. Their computer lab was located in part of the library, open to the main area with the computers arranged in circular groupings. These students were the only ones who had Macs with color capability and enjoyed trying it out.

Teacher C was also an experienced mathematics teacher, a mathematics major who had been teaching math classes, as well as other subjects, in a small town junior-senior school. He was comfortable with his students, having taught them previously and dealt with them also as the assistant principal of the school. He mentioned that he was interested in doing some computer work to supplement that which he was already doing with the graphing calculator.

Teacher C's class of 18 students was the only Mathematics 20 class offered in the school during that year. Their self-contained computer lab was in an area presided over by the Business Education teacher, but was available for booking by other classes.

Teacher D was teaching two Mathematics 20 classes with a total of 46 students at the time of this study, although his background was almost entirely in the sciences. He was now approaching the end of his second year of teaching. Teacher D was quite open to activities which got the students "...involved in their own learning". He had been using Zap-A-Graph™as a demonstration tool with his students during the semester.

Although Teacher D's school was located in a town of about 8000, its students were from a mix of rural and urban homes. It had a large computer lab as part of the Business Education Department, one section of which was devoted to Macintosh computers. These could be booked for other classes when and if space was available. The two Mathematics 20 classes taught by Teacher D accounted for about half of the Math 20 students for that school year.

Students Response to Using the Computer

After the students finished the unit, they were asked to complete a 10-part questionnaire. They were encouraged to be critical so that the materials could be revised for future students to use. (See appendix for "Evaluation Form for Students", pages 93 - 94). There were 105 student questionnaires turned in, although not all questions were answered by all students. Forty-one of these were from Mathematics 10 students and 64 were from Mathematics 20 students.

Student Background of Computer Use

The first question in the survey asked about age and gender, in order that the results of the other questions could be viewed in relation to these characteristics. Secondly, the students were asked about their use of computers in previous mathematics classes. Very few of them, three Math 10's and ten of the Math 20's (about 14% over all) remembered doing so. One of the classes had used graphing calculators previously, and another had a single computer available for class demonstration. A few had dim recollections of using the computer in elementary school. This was surprising, in that these classes were in school jurisdictions where computers were being made available, at least at the high school level.

The Mathematics 20 students came from a jurisdiction where special efforts had been made as many as ten years ago, to introduce LOGO to teachers. Computer labs were available in the junior high schools when these students attended there, but, if the students are to be believed, their mathematics teachers had not taken advantage of them. One possible explanation lies in the time required for adapting teaching practice to use a new method--to seek out software, to arrange (and in some cases fight for) the hardware, and to prepare lessons which use this tool effectively. Although there were places in the curricula and resources where computers might be used, this use is treated as an elective component of the programs, rather than a required one. Another explanation for the lack of computer use may be that the background of the teachers at elementary levels (and often junior high school, as well) did not include preparation in either computer use, or in teaching mathematics in a manner in which computers might be seen as a useful vehicle of instruction. This situation may be improving, but for the Math 10's and 20's in this study, computer use in mathematics classes was not common.

Very few students reported using the computer to program, and no one mentioned it as a problem solving tool in mathematics or science. Students did report a high degree of computer use in other subject areas at school. Eighty-three (close to 80%, 76% of Math 10's and 83% of Math 20's) mentioned using computers as a word processing tool in writing papers, usually in English classes, or were enrolled in computer related classes such as accounting, word processing, or desktop publishing. However, word processing, especially when it is used simply as an improved method of typewriting, does not require the same level of cognitive activity as programming, or activities which require interpreting what is appearing on the screen.

The results of computer use outside of school held some surprises. There appear to be many students who have limited experience with computers. Only fifty-one (about half) reported some degree of computer use outside of school. Those students who did mention using computers most often mentioned word processing uses, both at school and at home.

Playing games on the computer was specifically mentioned by 34% (14 out of 41) Math 10's, but of this group, nine were boys from Class B, three were boys from Class A, and two were girls. Of the Math 20's, 22% (14 out of 64) mentioned games, with a fairly even male/female split.

The Math 20 girls led the boys in computer use outside school, but not by a great deal (56% to 43% with 50% overall). The Math 10 boys reported about 75% computer use outside of class whereas the girls reported about 20% use. Only two made comments which suggested that they were serious computer users.

These results raise some questions for further consideration, but perhaps it is safe to conclude here that we should not assume that our students have extensive experience with computers. Some have almost no background, and will need extra help to overcome their computer anxieties. Those with word processing or game backgrounds may operate the software well, but will need to learn how to use a program as an exploratory tool, because neither of these applications promote a reflective approach to computer use. As Colgan (1993) suggests, "...'learning to learn' skills are pre-requisites to any technological implementation" (p. 1712-A).

Working With Partners

Students were often required to work with partners because of the number of available computers in the labs. As there are definite advantages in working with a partner (Hoyles and Sutherland, 1989), this was considered as something that would enhance, rather than detract from, the research. Most students worked with partners and felt that was satisfactory. One commented, "I like (to) work with a partner in the computers because if you make a mistake in your final result you can discuss and you know there is a mistake only comparing the results," while another concluded, "For the beginning, I worked alone, after I understood the basics it was better to work in groups and discuss changes in graphs". One who had worked both with a partner and alone commented, "...both ways worked quite well except I think I learned more on my own."

Of the 83 students who worked exclusively with a partner, only four felt this was not the best way to work. One commented, "I would do work, and he wouldn't. That bugged me. Maybe if I just had a more co-op. partner, it would be good" and another complained, "Working with a partner slowed me down. I knew what I was doing but was forced to explain everything to her." These students might have found more satisfaction if their partner had been chosen more carefully, or if the benefits of "teaching" others had been more clearly understood. The student who commented, "Working alone is better because you are more focused on your task with less distractions" did not see the benefit in discussing and negotiating meaning with a partner.

Only one of 16 who worked alone felt doing so was unsatisfactory. Even when working alone, there were frequent consultations among students. From these results, it appears that students can work satisfactorily in pairs; the situation is even better if there are enough computers so that students have a choice of working with a partner or alone.

Exploring Beyond the Assignments

Students were asked if they explored parts of the program other than those they were required to use. Slightly more than 50% answered "yes", but when separated by gender, the boys showed a greater tendency to explore. In the Math 10 group, 43% of the girls compared to 71% of the boys mentioned trying out different parts of the program. For the Math 20's, the results were 55% compared to 64%, based on a total of 59 students responding. These differences when results are examined by gender, both here and with the earlier questions, beg further study. (See "Conclusions").

One pair of boys tried out all the menus on Zap-A-Graph™with the "Family" option in an attempt to fill up the screen. (The "Family" option allows redrawing graphs an almost unlimited number of times with one parameter changed by a selected amount each time.) In the process of carrying out this challenge, they investigated the shapes of many new (to them) functions and developed some sense of how changing parameters in their equations would change the graphs produced. We encouraged this sort of exploration, but did not allow students to wander out of Zap-A-Graph™and into some of the other programs which were available to students of other classes.

Interpreting the Student View of "Learning About Lines"

Forty-one Mathematics 10 students were asked to rate the materials in "Learning About Lines" as either satisfactory or unsatisfactory in four areas. Almost all agreed they were easy to read, but 30% felt the procedures were not explained well enough so they could work on their own and almost the same number felt there were not enough examples. One student commented, "On worksheets you can put more examples how to do a question, instead of us asking you for help all the time,..." Another student complained that "There wasn't example like in the Math book." There were comments about the need for more introductory explanation and for better and easier instructions.

There were a few places in the original materials where there were misprints and small errors in the instructions. One obvious problem was that students were not accustomed to reading as carefully as they were required to in these lessons, so that sometimes they missed significant instructions. The most obvious of these was the one to "ERASE CURRENT GRAPH" which returns Zap-A-Graph™to its original equation before performing another transformation. To help with this, the materials were spread out more in later editions. There was an open-ended opportunity to create examples to explore, but we found that students seldom went beyond those required in the worksheets.

As with any form of instruction, there is a fine line between too much information (so that students ignore it anyway) and too little (so that they cannot proceed without asking for help). Each student requires and uses different amounts and types of explanation. Experience in designing exploratory lessons such as these, and student experience in working with such materials would solve many of the problems we encountered.

Students were asked what they found the best part of this method of learning to be. Many commented that it was fun, interesting, that it was faster, more accurate, and easier than working by hand. One commented, "The ability to see the line of the equation (was the best part). Not just plain letters and numbers. It helped imagination." Another mentioned that the best part was that the graph, equation, and table of values were available at the same time.

When asked for the worst part, some commented on the explanations and examples, as discussed earlier. Some mentioned that they didn't like having to erase their last examples in order to do a new one. (Zap-A-Graph™transforms the last graph drawn. If students do not return to the original graph each time, their values used for the transformations would not correspond to the numbers in the equation, so that the relationship between the graph and the equation is no longer obvious.) Several made comments like, "It let you take short cuts which caused problems when I had to do the work at home." and there were some comments like, "The computer isn't there when you take a test." (Discussion of these comments are included in the next section.)

When asked to choose the method they would use if they were a teacher planning this unit, about 50% chose methods involving computers. The class who had been using graphing calculators often made that their choice. Over 80% included either graphing calculators or computers among their choices.

Many of the comments at the end of the questionnaire were pleasant "warm fuzzies". Some referred to changes which needed to be made in the materials, or the method of presentation and have been included above. A few were more general, like the following: "I think that this class helps you to use computers for math and also helps you to understand computers which we all need in the future anyway."

Teacher A's class wrote an end of unit test which was adapted from the unit test usually given by the researcher at the end of this unit in her own regular classes. The class average was in the 60-70% range and judged by the classroom teacher to be in line with previous unit results.

Interpreting the Student View of "The Significant Seven"

When asked to respond to the materials in the units, again most students felt they were satisfactory. Dissatisfaction occurred most often with the number of explanations and examples provided. Interestingly enough, the girls expressed slightly more dissatisfaction the boys. The most extreme case was that of question C regarding the number of examples in the lessons, where 31% of the girls were dissatisfied, while only 3% of the boys were.

The Mathematics 20 students' comments regarding "The Significant Seven" were similar to those of the Math 10's when they were asked about the best part of the computer method: this method was fast, accurate and easy. Others responded that they found the ideas easier to comprehend, and that they got the main idea of the shapes of the graphs. They liked not having to make tables of values and hand-draw all the graphs that they got to see.

When asked about the worst part, many responded that they found it confusing to do similar questions on paper, especially in test situations. Some felt they had missed the basics because the experience moved too fast. One commented that this type of lesson "didn't make you think for yourself". As there was always the opportunity to redo activities or try more examples, and to reflect on results, these comments are disturbing. Perhaps students were simply unfamiliar with exploratory activites and will improve as they carry out more of them. Perhaps a change in the number and type of questions asked would be of help to these students. A few just didn't like working with computers. Certainly this unit could be made available in other ways to students who do not wish to use the computer, although the computer experience might be of benefit to them despite their negative attitudes.

Many students commented that "...it might help if there was a couple of classroom classes, so then we could sort of touch home." This idea was expressed often with both the 10's and 20's, so in the researcher's later classes, the format was changed to reflect these concerns. These comments reinforce the need for classroom discourse along with pencil and paper exercises as part of the teaching of this material. The students need help connecting what they see on the screen with the principles underlying the pictures.

When asked to choose a method of instruction, only about 10% did not choose the computer at all. Some of these students preferred graphing calculators; others wanted teacher explanation. One commented, "...I fail to see the point of this. I'm old-fashioned and don't like computers."

Most of students were very positive about the unit, like the student who said, "This was the most enjoyable math unit I have ever had. It was easy to understand, yet challenging. Well done!" Another commented, "This course was a merit in helping me to visualize graphs an their corresponding relations" and one commented "The computer made learning something complicated seem easy. There was no pressure to do more work than you could handle. It allowed you to "learn" concepts such as translations, by yourself." On the other side there were comments like, " I didn't learn much even though I went to every class." Perhaps the key phrase in this comment is "went to every class", which suggests that just being in the class should be enough to guarantee learning. Learning in this setting requires considerable personal initiative.

One interesting comment was this: "...I figure that alot (sic) of people are working harder than they should studying, when they should have been taught, and learnt." This corresponds with similar attitudes from many of the students encountered by the researcher in subsequent trials, the idea that the teacher isn't doing her job if the students have to search for their own learnings. Teaching using student exploration has not been common beyond primary classrooms (Yellowhead School Division Evaluation, 1994), so that the expectation these students does not include responsibility for developing their own ideas from structured activities provided by the teacher. If critical thinking is to be encouraged, students need to have the experience of searching for mathematical meaning under the guidance of their teacher, rather than expecting it to be delivered neatly packaged in every situation.

Other students commented that "If instruction is given on computer, tests should be on computer." The original intention was to use the computer labs as a way to develop ideas, which would be applied to paper and pencil work without the computer. Many of the traditional assessment questions become much easier if a graphing utility is made available during the assessment. For example, "How does the graph of the function expressed by $\mathbf{y = x^2}$ relate to the graph of $\mathbf{y = 3(x + 2)^2 - 5}$?" is a different question depending on whether or not graphing software is available. When it is asked without a graphing utility, students are required to predict the transformations from the equation. When asked with a graphing utility available, it tests the student's observation skills when asked to compare two graphs. Although both of these skills are important, the first is more demanding than the second, and more in line with the requirements of the curriculum. It is not surprising that the students wanted to use the computers for their test!

The concern regarding assessment is certainly a valid and one which is discussed in the NCTM documents. The Standards (1989) recommend increased use of "...calculators, computers and manipulatives in assessment"(p. 191). If assessment is to be aligned with the instruction (*NCTM Assessment Standards*, 1995), then students, at least in part, should be assessed on the way in which they use the computer software as a tool to explore functions. For this, the computer would be essential. If some objectives of the instruction require that students be able to work independently of the computer, then paper and pencil tests are still a reasonable part of the assessment. Simply allowing the computer to be used on a traditional test, as suggested by some of the students, would not solve the assessment problem.

An end of unit test was given to students in the three classes who used "The Significant Seven". The test was simply an adaptation of a paper and pencil test previously used with the researcher's Math 20 classes for the same content. The three class averages were 68%, 68% and 69%, and were judged quite satisfactory by the teachers involved.

The Teacher View of Using the Materials and Methods of the Study

Although there was a list of "Focus Questions for Teachers" (see Appendix, page 87), the interviews often took on different directions. Their comments fell into several general categories which are summarized and commented on in the sections which follow.

The Best and Worst of the Teaching Materials

Both Teachers C and D felt that getting the students to do some hands-on work with graphing was a strong point of this method of instruction. Students could see so many more graphs than either teacher or students could generate by hand. Teacher C noted that teaching with this method did not take as much time as he had expected, possibly because the students had some computer experience in other subject areas.

Teacher A felt that the best feature was getting the students involved in some computer activity, given that this is the era of technology. For him and his students, it broke the routine of blackboard and seatwork.

For Teacher A, the fact that students did almost no drawing of graphs on paper was a disadvantage. The idea "...that an ordered pair is actually a physical point, and then a group of these ordered pairs were part of a line" was one which students did not internalize. He recommended that revisions of the unit should include some more pencil and paper work, starting more slowly than we did in "Learning About Lines". Teacher D supported the idea of interspersing classroom work with the work in the computer labs.

Teacher D was also concerned about students working with partners. He mentioned the "...kids who piggyback, kids who aren't doing the work but just copy it and don't actually learn anything." This is always a concern for teachers whenever group work is attempted. Later in the discussion, he pointed out that many of the students he observed in pairs often were discussing math concepts. His second concern was for students who feared computers and would be helped by a partner. In the end, he seemed to conclude that the pairing had more advantages than disadvantages.

Teacher B mentioned problems with the worksheets in terms of format and explanations. Her perception that students often ignored the print material and simply followed the illustrations of screens has proven to be correct in many other classes as well.

One comment from Teacher C about the meaning of the term "sketch" led to some changes in presentation in later work. Originally, the intent was that sketching would be a very quick rendering of the graph, but after some of the students' work was observed, and Teacher C's comment noted, "sketching" became more precise. In later uses of these materials, the teacher has discussed techniques for sketching, such as using intercepts, and observing the manner in which the graph changes as it moves away from a reference point (such as the vertex of a parabola). These ideas emerge directly from the study of transformations and also help students when they are trying to determine the equation from the graph of a function.

Teacher C also asked for clarification of the intent of the written work. Will the written materials be subject to teacher assessment, or are they simply a student record of what was happening? He felt the students should know these things before beginning. These concerns have been noted as part of the general concern for improved assessment of the unit.

When asked about how well he felt students would retain the ideas from this unit, he commented that:

> The theory should be that it should stay with them longer having gone through the physical and visual process of seeing this happen and maybe understood it thoroughly. ...But I'm still of the old school enough to think that maybe my drilling and showing on the blackboard might impress them every bit as much in the long run. I really don't know. I'd be interested to find that out.

There are some difficulties in answering the question posed by Teacher C. Does this approach make any real, long term difference in student learning? To answer it we need to carry out both short and long term studies, with a variety of types of questions. More studies such as those carried out by Guttenberger (1991,1992,1993) and Ruthven (1990) would help us answer these questions. It is important that we attempt to provide reliable data on this key question; making computers available for mathematics classrooms is too expensive and the development of instructional models too time consuming, to be carried out without more evidence.

All four teachers interviewed liked the idea of classroom activities other than chalk and talk and so they were willing to consider computer graphing as a way of varying their instruction.. Teacher B mentioned being constantly on the watch for activities which were effective and fun to do. Teacher D mentioned that, "Sometimes it's nice to take that break and get away from talking and working at the blackboard." Teacher C agreed, ending with "...students like a change, too." This is one selling point for computer graphing: it provides variety for teachers both teachers and students.

Changing Teaching Practice - Why are teachers reluctant?

Given the enthusiasm shown by the teachers interviewed in this study for the use of computer graphing, and their willingness to try out something new, the question arises: "Why hadn't they been using the computer in some way before the study?" All had labs available, most had Zap-A-Graph™on the computer already. Teacher B had used the graphing calculator with her Math 10's, so was actively using graphing technology, but had not tried out the computer graphing. Teacher D had been using Zap-A-Graph™as a demonstration tool, but had not taken the students to the lab. Teacher C used graphing calculators for some purposes, but not extensively with Math 20's.

The element identified by all of the teachers in this study as critical to incorporating new practice is the idea of time. "You cause people to put extra into it (new practices) by giving them a little bit of time, time to pause and think and come up with a new idea that they want to try," (Teacher D). He found that being in the classroom full time absorbed all his time and energy, leaving little time for the development of new materials. Teacher C mentioned, "the thing that draws you back...is the time." These teachers talked about the shortage of teacher time for preparing new materials for use in the classroom.

Teacher B commented: "...ideas are super, they're great, I can think of 12000 of them myself, but it's can I fit them in?...not all, very few actually, that allow the kids to do learning at their own rate, the whole bit." In her case, "time" referred to instructional time and "fitting them in" referred to the limited time available to sufficiently address all the topics of the Mathematics 10 curriculum. In her view, any use of activity based instruction would take more time than direct teacher instruction, and would have to be weighed against projected results. Teacher C agreed, commenting, "...any investigative type of approach to me seems to be very time consuming," and that "..you have to weigh the time factor against the results." Teacher C did admit, however, to being surprised that our computer based instructional unit did not take any more time than the one by regular instruction would have.

"Risk" is another area of concern. Webster defines risk in part as "the chance of injury, damage, or loss". In this case, we would add "...to our professional or personal reputation or to our own sense of adequacy". Students, parents, and administrators are not always understanding when a new idea doesn't achieve expected results; teachers themselves are often self-critical. Teacher B commented,

"...lots of times you look like an idiot trying out something new, but it might be worth the risk,". For Teacher C, risk was not as much of a problem. He was working in a situation where he had established credibility, and frequently approached new problems with the "I don't know, let's investigate this together" attitude. Teacher A felt comfortable with using technology, but mentioned that likely it was fear of the unknown (technology) which held others back from embracing computers in their instructions. Teachers B, C, and D all mentioned that they would need to spend some time working with materials which were not their own before they would be comfortable using them in the classroom.

Changing Teaching Practice - What is the best form of professional development?

There has been much written about self-directed professional development in an attempt to encourage teachers to embrace new teaching practices. Unfortunately, the teachers are not reading or believing these articles and are constantly frustrated by inservice activities designed to lead them in this direction. Teacher C commented about a meeting both he and the researcher had attended: "I told *** at our math meeting that he could have just put it to us and we would have been well organized by now." In all fairness to Teacher C, he went on to comment that "...when you're going through some investigative thing trying to get input, it takes time to do a good job of that. It might be worth it, but then you have to weigh the time factor against the results."

Three of the interviewees were quite adamant that good seminars required enthusiastic, confident speakers who got their listeners excited about new ideas. Teacher C saw many professional development activities as giving him some "think time" and boosting "enthusiasm", while Teacher A felt that if properly structured as a good learning experience, a one-day seminar could be an effective way of showing teachers how to use an idea in the classroom. Teacher B agreed, saying, "An inservice where you really sat and did it as though you were the kid going through the program itself would be good." Teacher D was looking for materials to use in the classroom so was not enamoured of the philosophic approach; he preferred spending his time with actual materials rather than in sessions.

Teacher A felt that the leadership for professional development should come from the math consultants and department heads partly because, "...usually they [the ordinary day-to-day math teachers] don't have the time to search out these new ideas". In addition, he mentioned that the school board office is the place where new software (and other ideas, presumably) are directed, rather than to the schools.

"Collegiality", or sharing among colleagues, has been highlighted as an effective form of teacher professional development. Again, promoting this collegiatity among real teachers is not easy. The four teachers interviewed seemed quite willing to discuss teaching problems with colleagues. Teacher B reported that her staff formed "...a big, close department." They shared a physical space where there were teacher work areas as well as a small staff lunchroom. She reported that they often discussed how their classes were progressing and argued about ideas from the literature which frequently came into the department. Teacher B came from what appeared to be a similar situation, but there did not appear to be the same sharing or interaction among members of the department.

In Teacher C's case, there were very few math teachers on staff, and not a lot of time, so discussion with colleagues about mathematics was limited. Teacher D spoke of discussions within the science department, where he felt more rooted, as an everyday happening among the three teachers involved. He felt these were a "given"; that the administration expected the department head to help the new teacher get organized. (Interestingly enough, the official requirement of the department head in his school was only to supply the basic teaching materials, like texts and curriculum guides. The rest of the sharing occurred on the initiative of the members of the department themselves.)

None of these teachers recognized what we were doing together as professional development. Professional development came from outside, and was nicely packaged in terms of presentation or prepared materials. It was not teachers working together or sharing on their own initiative in an effort to improve practice. Is this simply a question of semantics, or are teachers ignoring the potential of collegiality as a means of changing their practice?

V. CASE STUDY

In the second phase of the study, students in the researcher's own mathematics classes used the materials which had been developed and used in the first phase. This chapter summarizes the experience of using these materials by the regular classroom teacher, rather than by a researcher who was a "new face" in the classroom.

Teacher and School Biography

Since graduating from the University of Alberta in 1965 with a B. Ed, I have spent about 20 years in the classroom. Although I believe strongly in the importance of following the curriculum so that students have the opportunity to meet the objectives set out there, I am also aware of the student voice, particularly when it calls for help. I like to bring meaning to lessons as I teach, and try to incorporate some activities in which students participate actively in their own learning.

Other than the one year spent at university in a Master's program, I have been teaching mathematics continuously in the only public high school in a town of about 8000 for twelve years. The school has about 500 students in Grades Ten through Twelve from both the town and the surrounding rural area. The computer labs are directed by the business education department. During the two years described here, the labs included 24 Macintosh computers (of which 20 might be operational at any given time) and over 50 IBM compatible computers. Depending on the use by the business education department, labs could be booked by other classes.

Updating and Materials

Part of the reason for choosing the topic of computer graphing in high school mathematics for my thesis was to prepare classroom materials which could be used in my own classes and those of others. During the initial phase of the study, when the materials "Learning About Lines" and "The Significant Seven" were developed and used with Mathematics 10 and 20 students, a number of issues arose which needed to be resolved.

These were:

1. Students needed practice and direction in verbalizing their observations. They needed to talk and write about what they had seen, after they have had the opportunity to make sense of it in discussion with others. For example, after students

experimented with the horizontal translation of a square root graph, they needed to talk and write about the effect of the translation. Filling in the chart correctly did not mean that they understood and could express what they had observed.

2. Students needed to practice in order to remember. Some of this practice should occur in a non-computer environment to ensure that they could operate without technology as well as with it. Their previous experience with functions was limited primarily to linear functions. They needed to have some practice in recognizing the shape of the graphs of other functions, so that they could connect the algebraic and graphical representations. They needed to talk about the features of each graph: domain, range, intercepts, and how the ordered pairs changed as the independent variable changed..

3. Students needed close monitoring to ensure that they filled in questions and charts in their materials workbook with care. The charts in the materials were designed so that students could record information to be used in drawing conclusions about the effect of the transformations being studied. For example, a chart summarizing the results of a vertical translation contained information about the old and new equations, and the change in the position of the graph. Such information was necessary in answering the next questions, which asked them to discuss what happened to the function under a vertical translation.

For many of these students, this type of independent learning was a new experience. They were accustomed to their teacher assuming full responsibility for presenting new ideas and documenting them with examples. Students did not always realize that the charts and questions had a purpose in developing the concept being studied so would rush through them in order to be "finished".

4. The concept of "sketching" the graphs as they were studied needed to be clarified for students in a way which helped them draw better representations of the graphs and helped them recognize the changes which occurred due to the transformations.

Using the Materials

Three classes of Mathematics 20 and one class of Mathematics 33 students used the materials ("The Significant Seven") developed in the first phase of the study with some minor modifications in the study of the part of the unit "Functions and Relations". In both curricula, students are required to investigate the effects of the parameters a, b, and c in the function **$y = cf(x - a) + b$** where f(x) can be a linear, quadratic, cubic, absolute value,exponential, or rational function.

Implementation

Most of the changes which were made were in the presentation of the materials rather than in the materials themselves. These changes were:

1. The students were booked into the computer lab for alternate classes during the time they were studying this section of the unit. While there, they studied one component of the graphs or their transformations. They answered the questions included with the labs, and sometimes were given exercises for which they used the computer. During the alternate classes they met in their regular classroom. We discussed the results of their computer investigations, carried out written exercises which required them to use their knowledge of the transformations, and wrote quizzes based on the topics completed.
2. Students handed in their lab assignments to be marked and those marks were included as part of their assessment on this unit. Knowing that their booklet of materials would be marked was intended to encourage a better quality of answer.
3. Specific instruction was included to encourage better sketching. Students were encouraged to examine how each graph changed as it left a reference point. (In all cases except the line, a reference point such as the vertex of the parabola, or the intersection of the asymptotes of the rational function is easily chosen. It can be repositioned to give a starting point for drawing a translated function. The effect of stretching or compressing the function is easiest to observe near the reference point so the behavior of the function in this part of the graph can be used as a guide when sketching new functions, or determining an equation for the graph of a function.)

Results

The majority of students involved in the case study completed the materials developed for use with Zap-A-Graph™ successfully and showed competence when answering questions regarding the functions and their transformations. At the end of the unit on "Functions and Relations" the class was given a paper and pencil test which included all sections of this unit as indicated in the Mathematics 20/33 Course Outline. One Mathematics 20 class achieved a class average of 61% (the final course average for this class was 59%), the second 56% (final course average of 66%), the third 70% (final course average 59%) and the Mathematics 33 class average was 72% (final course average of 54%). The tests given were different in each case, as were a number of other factors, such as the amount of review and the timing of the test relative to other events in the school.

The behaviour of the classes was typical of that during their regular classes. A few procrastinated, hoping that they could wait until the summary of results was provided and thereby avoid developing their own conclusions from the computer investigations. Others flew through the materials with minimal regard for the results of the explorations, then asked for help in filling in the questions requiring that they discuss their findings. One or two blamed every problem they had in their mathematics course on the computer. Most of the students accepted the experience as part of the course, and did their best to accomplish the tasks required of them.

The classroom sessions and the paper and pencil exercises helped to clarify the work done in the computer lab. Marking the assignments encouraged a more serious attempt to answer the questions, but students still tended to answer questions in a minimal fashion. For example, frequently the question, "Describe what has happened to the graph and the equation when this [vertical translation] was carried out," led to the answer "The graph goes up." Considering that many of these students were also students in English 20, and could express themselves well in written English, the problem was attributed to lack of practice in verbalizing ideas in mathematics and to being unaware of the extent of detail required to provide an acceptable answer.

There are phenomenal differences in the rate at which students are able to complete the lessons. For example, one hour was allowed for the lab on vertical stretch. Some students completed it in half an hour and went on to work in the textbook. Others were not finished at the end of the hour. In some cases, students were not finished because they were reluctant to take the activities seriously. In other cases, students were taking time to understand what they were seeing. Students could always arrange to put in extra time outside school hours, or during spare periods, but only a few did so.

The classroom discussions revealed the limited hold students had on the concepts of graphing in general, and the transformation ideas which they were exploring in the computer lab. *Students were novices when dealing with the concepts* of graph and equation, even though they have had some previous experience with lines and with graphing real-life relations. They knew that they could draw the graph by plotting ordered pairs from a table of values generated by the equation, but had trouble understanding that they could determine if a given ordered pair belongs to the function by substituting into the equation. They might spend an hour exploring horizontal translations quite successfully, but had lost track of their ideas by time the classroom discussion took place the next day.

This limited retention was illustrated vividly one day, when a student was beginning the first transformation activity. Moments before, she had completed the first computer exercise where all seven graphs are created on the screen, and was now asked to call up one of them on which to carry out a vertical translation. She complained that she had no idea how to do that, yet she had in front of her all the instructions to do so, and had drawn the very one required using Zap-A-Graph™just moments before.

Recommendations

Despite concerns which arose during the case study, using Zap-A-Graph™with the study materials developed is a viable method of presentation for the study of these units. Some of the concerns which arose during the case study can be addressed to improve the results when these materials are used in the future.

1. The materials can be rewritten to include more activities for students to carry out using the software. The original idea that students would explore on their own, once shown how, was not borne out in this research. This may be more due to the lack of experience with the techniques of searching out knowledge, and the lack of a strong mathematical base in the subject of functions from which to work, than to general reluctance on the part of the student.
2. The questions imbedded in the materials need to be reconsidered. These questions are included to enhance student reflection on the explorations. Most students in high school are not accustomed to taking time to think about their

mathematics lessons in much depth. Good questions can slow them down and help highlight important concepts.

There is a need to include more questions during the graphing of the seven original functions. Not only would this focus the students on each function individually, but it would provide the opportunity to connect these functions to other parts of the curriculum where they are already being studied. (For example, exponential growth is studied in a unit on "Radicals and Exponents" which includes examples of populations doubling. Revisiting this earlier discussion would strengthen the understanding of the exponential graph.)

Questions in both the materials and the discussions should include those which help students understand that all functions are not linear in nature. As teachers, we sometimes forget that much of the work our students have done up to this point is with linear functions. The transition to recognizing other patterns of change merits considerable discussion.

3. Every effort should be made during the study of functions to ask questions which lead into topics studied in Mathematics 30, particularly with the study of polynomials and trigonometry. It is not necessary to go beyond the Math 20 curriculum in order to prepare students for questions they will meet in the future. Writing activities, and the discussion of concepts like y-intercepts, zeros, domain, and range, lead into many concepts in Math 30 and beyond. If time is available, applying the transformations to the graph of $\mathbf{y = \sin \emptyset}$, which is introduced in the trigonometry unit in Math 20, would lead well into similar work in Math 30.

4. Students need to have the opportunity to spend extra time in the computer lab if they honestly require more time to complete the activities in the materials. Perhaps this means having a lunch-time session or two during the time in which this study is undertaken.

5. A new approach to assessment needs to be considered for this unit. Several factors should be considered. If technology is used to develop the ideas in the unit, does this imply that technology is required for the assessment? The materials had previously been tested by paper and pencil tests, which included topics from the entire Functions and Relations Unit, not just those sections of the unit included in the computer investigations. Calculators and computers were not used in the

assessment, because the computer activities were considered as labs such as those in the sciences.

In *Curriculum and Evaluation Standards for School Mathematics*, the NCTM advises that increased attention be given to using calculators and computers in assessment (1989, p. 191). If technology is allowed, then the tasks required of the students must be quite different than those presently used. Recognition of functions from their equations, and sketching a graph given the equation, become trivial activities when they can be typed into a graphing utility.

One solution to the problem is to create an assessment framework which includes some activities to be carried out on the computer, and some to be carried out with only paper and pencil. Some of the tasks should require that the student apply what has been learned in this unit to the study of a function not yet studied. Other activities should require written answers which use the vocabulary and information learned in this unit to explain a situation presented to them. In addition to the formal assessments, student daily work should be reflected in the assessment. The labs themselves, the textbook assignments, and the quizzes all have a place as part of the final assessment.

Reflections on the Case Study

The case study grounded the research in everyday classroom experience. Knowing the students made me more aware of how they were reacting to the tasks I had set. Because they knew me, they did not refrain from expressing frustrations or try to be extra nice because I was a visitor. Most of them proceeded through the materials without incident and demonstrated that they had learned what was required on a number of tasks including a unit exam. Some students enjoyed the freedom of being able to proceed on their own; others were not so enthusiastic and would have preferred a teacher-developed classroom lesson.

Some, however, choose to make the computer a scapegoat for poor performance. One group complained that they would have done well on the unit test (which included topics from the entire unit) if it hadn't been for the computer section. Upon checking their test papers, it became clear that their performance on questions from the topics studied on the computer was actually better than on questions from the other topics. Unfortunately, these sorts of comments mask the real situation. Did these students have a real problem with the computer-based study, or were they simply seeking to explain a poor result by blaming the "new" method?

Changing Teaching Practice

My own experience has helped me understand why teachers, myself and others, are reluctant to change practice. In some cases, the costs are too high when compared to the improvement in return. In my case, cost almost always translates into demands on my own time and energy. Loss of credibility with my students, administration, or the school community is another possible cost.

The Mathematics 10 Experience

With the Mathematics 10's, I have been reluctant to change my practice to include using the computer to any extent because I have not felt that the risk was worth the return. When I returned to the classroom after my leave, I was assigned, among others, two rather large and vocal Mathematics 10 classes. They had a very traditional view of mathematics teaching, and that view did not include searching for their own meaning.

This reluctance to give up old ways of learning is mentioned by T. J. Bassarear (1987) when he discusses obstacles confronting teachers trying to use constructivist methods of instruction (p. 263). I was trying out some of the new ideas encountered in my year of study, forgetting "the construction of new ways of learning need time to develop"(Bessarear, 1987, p. 265).

The more outspoken students made comments such as "I wish we could have Mr. *** to teach us. He knew what he was doing," or "Why don't you just tell us how to do this?" when I would seek to give some meaning to the activity at hand. Although the approach used in "Learning About Lines" had much to recommend it, I chose to limit the computer exploration to an activity on slope and y-intercept which could be carried out on computer or with graphing calculators. This was more comfortable for the students, who were having serious difficulties adjusting to the demands of the Mathematics 10 curriculum and to a different way of teaching.

Professional Development

Since returning to teach two years ago, I have been able to convince some of my colleagues to try out "The Significant Seven", at least in part, with Math 20's and 33's. Those who have done so will likely continue to use the computer for these topics if feasible, but will adjust the approach to their own teaching style. I consider this a successful example of collegiality as a form of professional development. In addition, I have offered three short sessions for other teachers in which these materials were featured.

VI. SUMMARY AND DISCUSSION

Restating the Problem

The intent of this study was to create and use computer graphing materials for teaching suitable units in high school mathematics, in particular, co-ordinate geometry topics in Mathematics 10, and transformation of several functions as required in Mathematics 20. In addition, the teachers who agreed to use the materials were interviewed as to the merits of the materials, and their opinions regarding how new ideas could be passed on effectively to teachers.

The Materials and Their Presentation

Generally speaking, both students and teachers agreed that the teaching materials and methods used to present them were interesting, and a good way to present the ideas. Both mentioned the need for more examples and explanations, more like the textbook, but for the most part were able to understand what was required.

Presentation

Originally, the researcher thought that all the concepts required could be dealt with in the computer lab, but it was quickly discovered that there were too many missed opportunities if the classroom experience was not included. It was necessary, however, to make the activities to be carried out in the computer lab self-directing, as the physical setting in many computer labs is not that of a self-contained, quiet classroom. In addition, the computer itself distracts the student from any instruction which might be happening. The teacher becomes a consultant as the student carries out the instructions given in the printed materials.

The researcher has often observed that when using printed instructions, students prefer to jump from illustration to illustration, neglecting the written instructions entirely. Some of the difficulties with instructions could be remedied by more careful placement of illustrations and text. The first edition of the materials was crowded and had too many steps on each page. Students required more examples to explore; they did not take full advantage of the opportunity to create and explore their own examples.

Questions

For the students in this study, this was a first-time experience in a kind of learning which placed more emphasis on determining their own conclusions rather than conclusions from the teacher or the text. Bassarear (1987) mentions that students often resist functioning at a higher than usual cognitive level. As teachers use more open ended activities and become more concerned about developing higher order thinking skills, their students should become increasingly capable of using investigative techniques.

Discussion

Both teachers and students indicated the need for more classroom discussion and explanation. For that reason, when the lesson materials are used now, a good deal of time is spent in the classroom, transferring what has been learned on the computer to non-computer mode. Some time is spent in discussing the experiences in the computer lab activities. Some is spent in summarizing, practicing, and in writing. For example, the following assignment has been used with Math 20 students:

> Mary and Frank are discussing how the graph of $\mathbf{y = x^2}$ has been transformed to become $\mathbf{y = 2(x - 4)^2 + 3}$. Mary says that the graph has become thinner and has moved 4 down and 3 right. Frank thinks it has become wider and has moved 4 left and 3 up. Are either Mary or Frank correct? Discuss their solutions and identify the correct answer.

Not only does this exercise reinforce the ideas from this unit, but it provides practice in writing solutions to similar questions frequently posed at the Mathematics 30 and beyond.

There are risks inherent in classroom discussions following investigative activities. Any discussion involves only a few of the 25 or 30 students in the class. Some students learn that, if they wait long enough, they will be able to copy down the summary which the teacher prepares during the discussion. Thus, they avoid the struggle to create their own meaning, and of course, miss out on the benefits of forming and testing their own conjectures.

Working In Pairs

Working in pairs was quite acceptable to most students. This is fortunate, as most classes are too large for the available computers in the average lab! It is desirable that students be given some choice in the matter, however, as the students indicated that not everyone was happy as part of a pair. Most students, even if not part of a pair, will discuss results with other students; this learning method seems to promote cooperation.

Variety in Instructional Practice

One very practical reason for using the computer graphing approach to relations and functions as presented in this study is simply the variety which is introduced into the daily routine. Both teachers and students often fall into the pattern of "chalk and talk" lessons. While the students are in the computer lab, the teacher can interact with the students individually while they are working on the lessons. Sometimes the student and teacher change roles, as many students are more accomplished in computer use than their teacher. Varying instructional methods can be revitalizing.

Changing Teacher Practice

The teachers involved in the research had a professional concern in finding the best and most efficient way to teach their students. Unless they felt that a new idea would improve instruction in their classroom, they would not try it out.

How Teachers See Professional Development

When asked about good professional development activities, these teachers spoke often of situations where others presented an idea to them. Speakers were expected to be dynamic, or the experience could be a waste of time. Teachers did not voluntarily mention forms of self-directed professional development. These teachers were quite willing to allow the research to be carried out in their classrooms, and were interested in learning from it. They could foresee adapting the materials for their own use, meshing them with their own teaching style. They did not include activities such as this in their definition of professional development.

The NCTM Professional Standards (1991) states that "As professionals, teachers take responsibility for their own growth and development". It may be some time before most teachers grow into that statement and feel empowered to seek out professional development on their own terms.

Fullan (1991) speaks of the costs of trying new innovations, and these teachers have echoed that idea. We must bear in mind that there are many innovations being showcased at any one time, not only in the field of mathematics education, but in education in general. Teachers must sort through a minefield of potential theories, and choose those which offer the greatest benefit for the least cost.

What Are Teachers Telling Us?

If we want teachers to use the numerous ideas presently circulating in mathematics education, we need to study the most effective ways of getting them involved. This researcher is firmly convinced that the most effective use of researcher and consultant time is with teachers in the classroom, or in facilitating the preparation of classroom ready materials which demonstrate classroom applications of "good ideas". Most classroom teachers do not have time to read the literature and spin out the ideas into classroom segments. They will take prepared materials, and reform them to suit their own instruction. They will extend these ideas, having been shown the first step and many will share good ideas among themselves.

Getting to the teachers with these ideas is another challenge as many do not belong to organizations like NCTM and MCATA. For some, a change in the curriculum or the diploma exam is a way to get their attention. For others, a new textbook can cause major changes in practice. For this reason, the role of the Department of Education curriculum developers, the authors of mathematics resource materials, and the universities continue to be critical in spreading new ideas.

Potential for Future Research and Development

There is great potential for future research in the areas studied in this thesis. A quantitative study, with a control group, and with follow-up studies after several months would give a more definitive answer as to whether this form of instruction improves learning and retention, and for which students. There are some interesting possibilities for gender studies: Do females benefit from a more visual approach to the study of functions? Do computers help or hinder females (or males) when used as an investigative tool? Are the differences in responses by gender significant, or due to the small numbers involved?

Another area requiring further study is the development of curriculum. Curriculum builders need to consider continuity when developing future programs. Teachers have a tendency to look at the curriculum horizontally-(Mathematics 10, followed by 20, followed by 30 (or 13-23-33)) rather than to follow topics vertically (develop a topic throughout the junior and senior high school programs). This vertical flow should be more evident in curriculum and textbooks. Presently, these are written often as discrete units by different authors, to fit a time frame which is driven grade by grade. Teachers, particularly those who have not had the opportunity to teach at several levels, are not always aware of natural flow of topics.

Relations and functions are an example of this discontinuity. In Mathematics 10, students concentrate on studying the line, but do not talk about it as a relation. Most developments (except the one used in the materials developed for this study) do not consider the line in the language of transformations. In Math 20, the line is reconsidered as a function, and transformed along with other functions. In Math 30, trigonometric functions are transformed in the same manner as those in Math 20, although the curriculum documents and text do not make a point of using the similarities to make the development easier. In Math 30 as well, the conic sections are examined under a general equation for quadratic relations. Although transformed conics are studied, the connection with previous work in transformations is not evident.

If students begin computer analysis of lines in Math 10, and continue with the functions in Math 20, they should be better prepared to understand functions and relations as they are met in future courses. This researcher has observed improved understanding of trigonometric graphing and polynomial functions by students who have worked with computer analysis of functions in Mathematics 20. More importantly, the students will have learned a new way of learning for themselves.

Summary

The main direction of this study was to develop and test classroom materials in which Zap-A-Graph™was used by students as a tool for exploring relations and functions. Using this as a base, the researcher examined the broader topics of computer use by students, and the professional development of teachers.

The findings indicate that it is possible to create lessons for students to use effectively in the study of relations and functions, but that it is the manner in which these lessons are used which determines their success. The research has also emphasized that for research findings to become practice, there must be considerable energy spent in facilitating their use in the classroom.

BIBLIOGRAPHY

Alberta Education . (1989). *Senior High Mathematics 10/13/14 Interim teacher resource manual.* Edmonton, AB: Author.

Alberta Education . (1990). *Senior High Mathematics 120/23/24 nterim teacher resource manual.* Edmonton, AB: Author.

Apple, Michael W. (1992). *Do the Standards go far enough? Power, Policy, and Practice in Mathematics Education. Journal for Research in Mathematics Education. Vol 23, No. 5,* (412 - 431).

Barrett, Gloria, & Goebel, John. (1990). The impact of graphing calculators. In Thomas J. Coomey. (Ed.). *Teaching and Learning Mathematics in the 1990's.* (pp. 205 - 211). Reston, VA: National Council of Teachers of Mathematics.

Bassarear, Thomas J. (1987). Obstacles confronting the implementing of constructivist instruction and three models for addressing these obstacles. In J. C. Bergeron, N. Herscovics, & C. Kieran (Eds.), *Proceedings of the Eleventh International Conference, Psychology of Mathematics Education, Vol. III,* (pp. 262-268). Montreal.

Battista, Michael T. (1990). Spatial visualization and gender differences in high school geometry. *Journal for Research in Mathematics Education. Vol. 21, No. 1.* (47 - 60).

Belenky, M. F., Clinchy, B. M., Goldberger, N. R., & Tarule, J. M. (1986). *Women's ways of knowing.* New York: Basic Books.

Bennett, J. Phillip. (1991) Effectiveness of the computer in the teaching of secondary school mathematics: Fifteen years of review of research. *Educational Technology 31.* (44 - 48).

Blackett, Norman. (1989). Using computers to encourage versatile learners. *Mathematics In School,* November, 1989. (17 - 19).

Bloom, L., Comber, G., & Cross, J. (1986). Use of the microcomputer to teach the transformational approach to graphing functions. *International Journal of Mathematics Education in Science and Technology, Vol 17, No. 1.* (115 - 123).

Cobb, P., Yackel, E. & Wood, T. (1992). A constructivist alternative to the representational view of mind in mathematics education. *Journal for Research in Mathematics Education, Vol 23, No. 1.* (2 - 33).

Colgan, L. E. C.. (1993). Graphing tools in the mathematics classroom: Some factors affecting successful use. *Dissertation Abstracts International, Vol. 54, No. 5.* (page 1712-A)

Collis, Betty. (1987). Sex differences in the association between secondary school students' attitudes toward mathematics and toward computers. *Journal for Research in Mathematics Education, Vol. 18, No. 5.* (pp. 394 - 402).

Corbitt, Mary Kay. (Ed.). (1985). The impact of computing technology on school mathematics: Report of an NCTM conference. *Mathematics Teacher, 78(4).*

Davison, D. & Pearce, D. (1988). Using writing activities to reinforce mathematics instruction. *Arithmetic Teacher. 36 (April 1988),* (pp. 42 - 45).

Demana, Franklin, & Waits, Bert K. (1990). Enhancing mathematics teaching and learning through technology. In Thomas J. Cooney. (Ed.). *Teaching and Learning Mathematics in the 1990s.* (212 - 222). Reston, VA: National Council of Teachers of Mathematics.

Demana, Franklin, & Waits, Bert K. (1990). The role of technology in teaching mathematics. *Mathematics Teacher* 83 (1) (27 - 31).

Dreyfus, Tommy & Halevi, Tirza. (1990). QuadFun--A case study of pupil computer interaction. *Journal of Computers in Math and Science Teaching, Vol. 10(2).* (43 - 48).

Dugdale, S., Wagner, L., & Kibbey, D. (1992). Visualizing polynomial functions: new insights from an old method in a new medium. *Journal of Computers in Mathematics and Science Teaching 11,* (123 - 131).

Fey, James. (1989). Technology and mathematics education: A survey of recent developments and important problems. *Educational Studies in Mathematics, 20.* (237 - 272).

Fullan, M. G. (with Stiegelbauer, S.). (1991). *The new meaning of educational change*. New York, NY: Teachers College Press.

Heid, M. Kathleen. (Ed.). *Algebra in a Technological World.* (1995). National Council of Teachers of Mathematics. Reston, VA: Author.

Hoyles, Celia & Sutherland, Rosamund. (1989). *Logo mathematics in the classroom.* London: Routledge.

Kelly, B. et al. (1987). *Mathematics 10.* Don Mills, ON: Addison-Wesley

Kelly, B., Alexander, B. & Atkinson, P. (1990). *Mathematics 11.* Don Mills, ON: Addison-Wesley .

Kieren, T. E. (1968). The computer as a teaching aid for eleventh grade mathematics: A comparison study. Doctoral Dissertation.

Lichtenstein, B., McLaughlin, M.W. & Knudsen, J. (1992). Teacher empowerment and professional knowledge. In Lieberman, A. (Ed.). *The Changing Context of Teaching*. Ninety-first Yearbook of the National Society for the Study of Education, Part 1. Chicago, Illinois: The University of Chicago Press.

National Council of Teachers of Mathematics. (1991). *Professional Standards for Teaching Mathematics*. Reston, VA. Author.

National Council of Teachers of Mathematics. (1995). *Assessment Standards for School Mathematics*. Reston, VA. Author

National Council of Teachers of Mathematics. (1989). *Curriculum and Evaluation Standards for School Mathematics*. Reston, VA. Author.

Papert, Seymour (1980). *Mindstorms: children, computers, and powerful ideas.* New York: Basic Books.

Rizzuti, Jan Marie. (1991). Students conceptualizations of mathematical functions: the effects of a pedagogical approach involving multiple representations. *Dissertations Abstracts International, Vol. 53 No. 10.*

Ruthven, Kenneth. (1990). The influence of graphic calculator use on translation from graphic to symbolic forms. *Educational Studies in Mathematics 21:* (431 - 450).

Schofield, J. W., & Verban, D. (1988) Computer usage in the teaching of mathematics: issues that need answers. In Grouws, D. A., Cooney, T. J., & Jones, D. (Eds.). *Perspectives on Research on Effective Mathematics Teaching, Volume 1.* Reston, VA: Lawrence Erlbaum Associates and the National Council of Teachers of Mathematics.

Shumway, Richard. (1990). Supercalculators and the curriculum. *For the Learning of Mathematics 10, 2.* (2 - 9).

Skemps, Richard R. (1987). *The Psychology of Learning Mathematics.* Hillsdale, NJ: Lawrence Erlbaum.

Skemps, Richard R. (1989). *Mathematics in the primary school.* London: Routledge.

Tall, David. (1983). Introducing algebra on the computer: Today and tomorrow. *Mathematics in School, November, 1983.*

Tall, David. (1985). The gradient of a graph. *Mathematics Teaching, 111.* (pp. 48 - 52).

Tall, David. (1987). The reality of the computer in the secondary mathematics classroom. *Mathematics In School, January, 1987.*

Tall, David. (1989). Concept Images, Generic Organizers, Computers, and Curriculum Change. *For the Learning of Mathematics , 9 (3).* (pp. 37 - 42).

Tall, David et al. (1984). The mathematics curriculum and the micro. *Mathematics In School.* September, 1984. (7 - 9).

Tall, David & Thomas, Michael. (1991). Encouraging versatile thinking in algebra using the computer. *Educational Studies in Mathematics 22.* (pp. 125 - 147).

Waits, Bert K. & Demana, Franklin. (1988). Microcomputer graphing: a microscope for the mathematics student. *School Science and Mathematics, Vol. 88 (3).*

Waits, Bert K. & Demana, Franklin. (1988). Manipulative algebra--the culprit or the scapegoat. *Mathematics Teacher, 81(5).* (pp. 332-334).

Wenzelburger, Elfriede. (1992). The learning of trigonometric functions in a graphical computer environment. *Proceedings of the 16th international conference for the psychology of mathematics education, 3.* (pp. 106 - 113).

Wenzelburger, Elfriede. (1992). The learning of trigonometric functions in a graphical computer environment. Unpublished paper, Universidad Nacional autónoma de México. (1992).

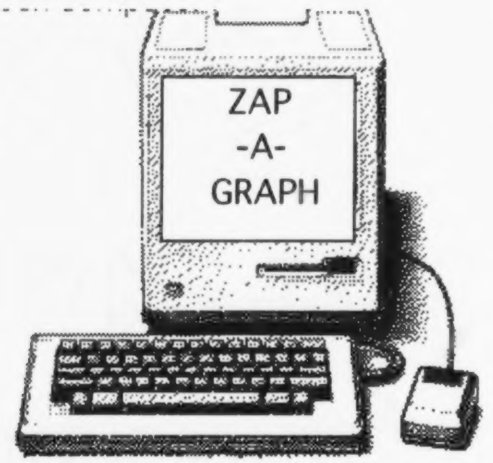
Wenzelburger, Elfriede. (1993). A learning model for functions in a computer graphical environment. Unpublished paper presented at NCTM (Nashville).

Yellowhead School Division No. 12. (1994). *Mathematics Program Review K - 12.* Edson, AB: Author.

APPENDIX

LEARNING ABOUT LINES

MATHEMATICS 10



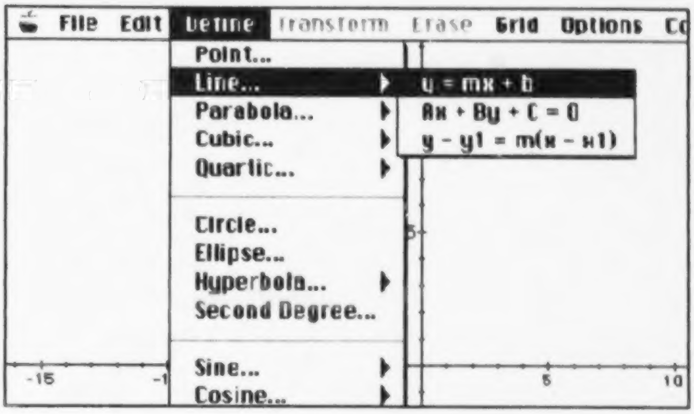
MARIAN OBERG
APRIL, 1993

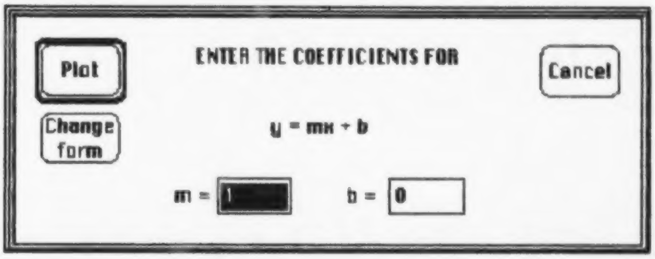
TRANSFORMING Y = X USING STRETCH

A. GRAPHING THE RELATION

Pull down DEFINE and click on LINE.

Choose y = mx + b.


File
Edit
Define
Transform
Erase
Grid
Options
Point...
Line...
Parabola...
Cubic...
Quartic...
Circle...
Ellipse...
Hyperbola...
Second Degree...
Sine...
Cosine...
y = mx + b
Ax + By + C = 0
y - y1 = m(x - x1)
-15
5
10


Plot
Cancel
Change form

ENTER THE COEFFICIENTS FOR

$y = mx + b$

$m = 1$ $b = 0$

Make sure $m = 1$ and $b = 0$.

To change the numbers, use TAB to move between the values.

Press ENTER when ready or click PLOT.

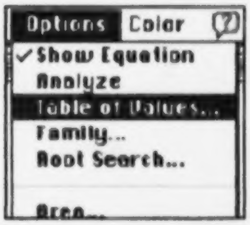
????QUESTIONS????

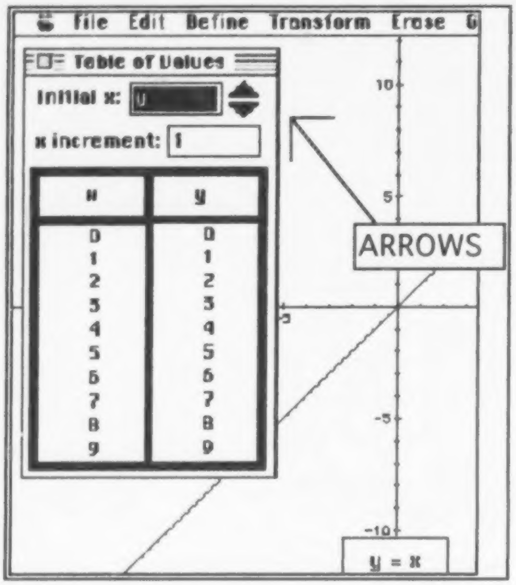
*What quadrants does the line pass through?..

*What is the x-intercept of the line?(Where does the line pass through the horizontal axis?....................................

*What is the y-intercept of the line?(Where does the line pass through the vertical axis?...................................

Pull down the OPTIONS menu and click on TABLE OF VALUES.


Options
Color
✓Show Equation
Analyze
Table of Values...
Family...
Root Search...
Area...


File Edit Define Transform Erase G
Table of Values
Initial x: 0
x increment: 1
ARROWS
10
5
-5
-10
y = x

x	y
0	0
1	1
2	2
3	3
4	4
5	5
6	6
7	7
8	8
9	9

Click on the arrows to change the values of x.

????QUESTIONS????

*When x = 3, what value does y have?.........

*When x = -5, what value does y have?.........

*When y = 13, what value does x have?........

Look at the table of values.

*How much do the values in the x-column change each time?...........

*How much do the values in the y-column change each time?...........

(TAB) until (Initial x:) is highlighted. Type in 25.

* How does this change the table?...

(TAB) until (x increment:) is highlighted. Enter 2 .

*How does this change the x and y values in the table?...

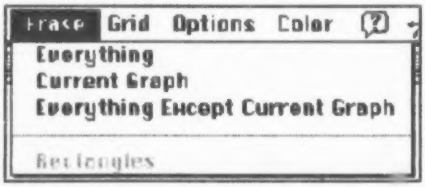
....

*What values would you enter to have the value of x in the middle of the table at 3 and change by 0.5 each time?...

Pull down (ERASE)

and choose

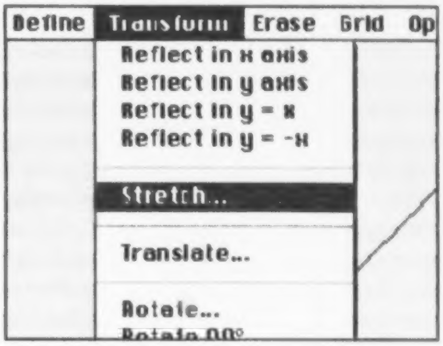
(EVERYTHING).

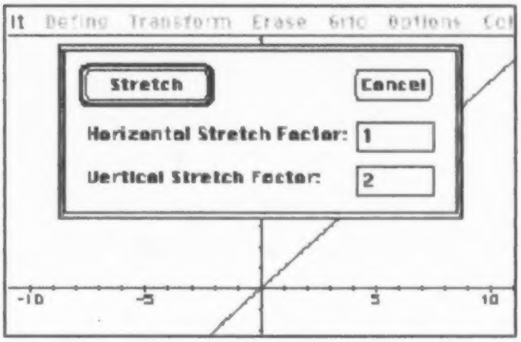

Erase Grid Options Color ?
Everything
Current Graph
Everything Except Current Graph
Rectangles

B. TRANSFORMING THE RELATION WITH VERTICAL STRETCH

Graph y = x (Refer back to Part A if you have forgotten how!)

Pull down the TRANSFORM menu and click on STRETCH.


Define
Transform
Erase
Grid
Op
Reflect in x axis
Reflect in y axis
Reflect in y = x
Reflect in y = -x
Stretch...
Translate...
Rotate...
Rotate 90°

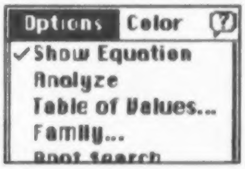

It
Define
Transform
Erase
Grid
Options
Co
Stretch
Cancel
-10
-5
5
10

Horizontal Stretch Factor: 1

Vertical Stretch Factor: 2

Use TAB to move to Vertical Stretch.
Enter "2".
The screens should look like the one at the left.

Pull down OPTIONS to make sure SHOW EQUATION is checked. If not, highlight and click to activate.


Options
Color
?
✓Show Equation
Analyze
Table of Values...
Family...
Root Search

*What is the equation of the new graph?

....

*Turn on TABLE OF VALUES.

As x changes by 1 unit, how much does y change? ....

Pull down ERASE and click on CURRENT GRAPH.

You will return to the graph of y = x, ready for the next stretch.

Pull down the TRANSFORM menu and enter a vertical stretch factor of 3.

*What is the new equation?..................................

*How has the graph changed from the previous one?

....

*Look at the table of values. As x changes by 1, what is the change in y?..

ERASE the CURRENT GRAPH.

Try a vertical stretch factor of -1.

*How has the graph changed? How has the equation changed?

....

....

Continue to stretch the line. Fill in the chart below.

REMEMBER: ERASE CURRENT GRAPH AFTER EACH EXAMPLE. ZAP-A-GRAPH ALWAYS ACTS ON THE MOST RECENT GRAPH. MAKE SURE THAT GRAPH IS ALWAYS Y=X.

VERTICAL STRETCH	EQUATION	CHANGE IN Y WHEN $\Delta X=1$	VERTICAL STRETCH	EQUATION	CHANGE IN Y WHEN $\Delta X=1$
2			-1		
3			-4		
5			$-\frac{1}{3}$		
$\frac{1}{2}$			$-\frac{2}{5}$		
$\frac{3}{4}$			$-\frac{2}{3}$		

C. DISCUSSING THE TRANSFORMATION

*Describe what happened as you changed the vertical stretch factor. Talk about the change in the position of the line, the change in the equation, and the change in x.

....

....

....

*To have the graph of y = 4x appear, we could:

1. *Graph $y = x$ and apply a vertical stretch of 4.*

or 2. *Enter $m = 4$ in the LINE menu.*

Make the graph of y = 4x appear on your screen.

Use the mouse to position the arrow so that it points to the line. Click and hold.

You will see crosshairs on the screen. Beside them, you will see the value of the pair where the crosshairs are located.

If the crosshairs are on the line, you will see a pair which belongs to the relation y = 4x.

(Note: it is quite difficult to place the crosshairs exactly on the line. Unlike the graphing calculator, this program does not TRACE.)

*What are two other ways to find out which points are on the line?

1. ....

2. ....

For the equation y = 4x, fill in the pairs which belong to the relation:

(1,) (2,) (3,) (-1,) (0.5,)

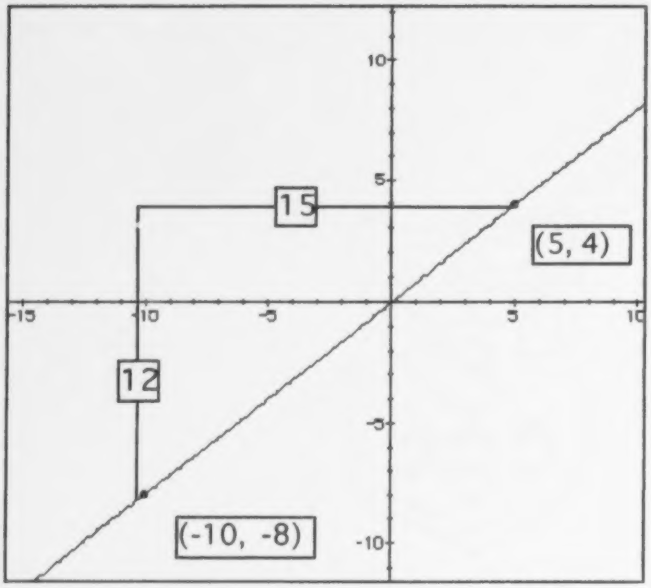
*A relation is graphed and the table of values is shown. Two entries in the table are (3, -9) and (4, -12). What is the equation?

....

*A relation is graphed and two points on the line are found to be (-1, 5) and (2, -10). What is the equation?

....

THE RATIO $\frac{\text{RISE}}{\text{RUN}}$ IS GIVEN THE NAME SLOPE.



On the graph at the left, the value of the RISE tells us how far we go up or down until we are across from the second point. The value of the RUN tells us how far we have to move across until we are back on the line.

We use + numbers for movements UP or RIGHT.

We use - numbers for movements DOWN or LEFT.

For the example above, the slope would be $m = \frac{12}{15}$ or any ratio equivalent to it.

*Name three ratios equivalent to $\frac{12}{15}$: ______, ______, ______

THE GAME

You'll need a partner for this one. Here's how it goes:

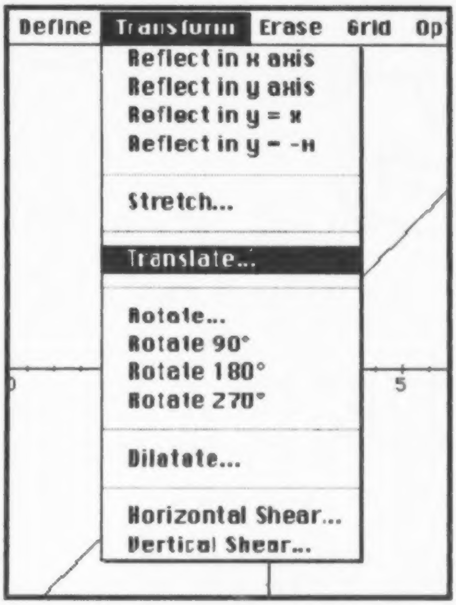
1. Player 1 uses (DEFINE)(POINT) to put a point on the Zap-a-graph screen.
2. Begin with points in Quadrant 1 until each player has 2 points.
3. Player 2 tries to draw a line through that point, using (DEFINE)(LINE)
4. If successful, Player 2 gets a point.
   If not, Player 1 may try, and gets a point if successful.
5. Take turns until somebody gets a point. The player who loses the round must start the next.

TRANSFORMING Y = X USING TRANSLATE

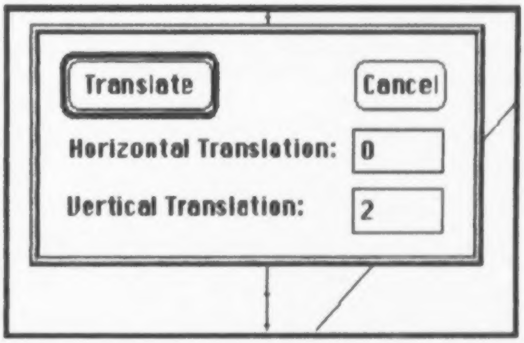
A. GRAPHING THE RELATION

Graph y = x on Zap-a-graph. (If you need instructions, see the first lesson.)

B. TRANSFORMING THE RELATION


Define
Transform
Erase
Grid
Op
Reflect in x axis
Reflect in y axis
Reflect in y = x
Reflect in y = -x
Stretch...
Translate...
Rotate...
Rotate 90°
Rotate 180°
Rotate 270°
Dilatate...
Horizontal Shear...
Vertical Shear...
5

Pull down TRANSFORM . and click on TRANSLATE .


Translate
Cancel

Horizontal Translation: 0

Vertical Translation: 2

Enter 0 for Horizontal and 2 for Vertical.

*Describe that you see on the screen

by answering the following questions:

1. Which **quadrants** does the new line pass through?

2. What is the **x-intercept** of the new line?

3. What is the **y-intercept** of the new line?

4. What is the **equation** of the new line?

5. What is the **slope** of the new line?

6. What is the term used to describe the two lines on your screen?

ERASE CURRENT GRAPH.

Try a vertical translation of - 2.

*Describe the change in the graph. Use questions 1-5 above as a guide.

1. ____
2. ____
3. ____
4. ____
5. ____

ERASE CURRENT GRAPH

Carry out **vertical** translations to fill in the chart below.

REMEMBER TO ERASE EACH TIME SO THAT YOU ARE TRANSLATING Y = X.

VERTICAL TRANSLATION	NEW EQUATION	Y-INTERCEPT OF NEW GRAPH
3		
-5		
$\frac{1}{2}$		
0		
-2		

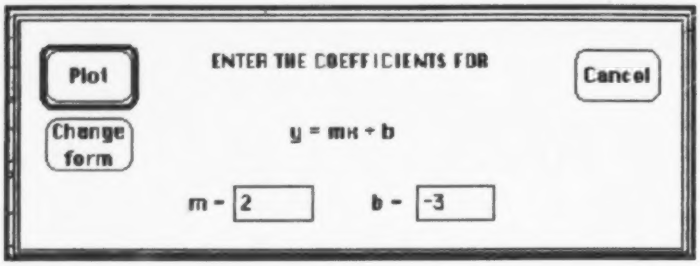
A **hypothesis** is a statement which we think to be true and which we make to explain what we think is happening.

*What is your hypothesis about the effect of vertical translations on the line $y=x$? How is the equation related to the translation?

____

____

EXERCISE



ENTER THE COEFFICIENTS FOR

$y = mx + b$

m = 2 b = -3

Graphs can be drawn using the LINE menu. For the first question below, enter 2 for m and -3 for b as shown. (m is always the coefficient of x, no matter where the term appears.)

What do the graphs of the relations in each question below have in common? What is different about them? Try to predict what will happen before you graph them.

1. $y = 2x - 3$
   $y = 1 + 2x$
   $y = 2x$

2. $y = -x + 1$
   $y - 1 = 4x$* (Needs work first!)
   $y = 0x + 1$

3. $y = \frac{3}{4}x - \frac{1}{2}$

   $y = 0.75x + 3.5$

   $y = \frac{3}{4}x - 1$

4. $y - 2.5 = 0.25x$
   $y = 4.1 + 0.25x$

   $0.25x - y = -1.5$

5. $y = -x - 6$
   $x + y = 2.5$
   $y = 4.5 - x$

6. $2x + y = 3$
   $y = 3 - 0.5x$
   $4.5x + 3y = 9$

TRANSFORMING Y = X
USING VERTICAL STRETCH AND TRANSLATION

How do the following transformations compare:

A. Graph $y = x$ using Zap-a-graph.
Vertical stretch by factor of 2.
Translate vertically by 1.

New equation:________________

B. Graph $y = x$ using Zap-a-graph.
Translate vertically by 1.
Vertical stretch by factor of 2.

New equation:________________

Factored form: ________________

Is the result the same? ________

1. Which of the following pairs of equations will give the same results?

A. $y = 2x + 3$
$y = 2(x + 3)$

B. $y = 3x + 1$
$y = 3(x + 1)$

C. $y - 4 = 2x$
$y = 2(x + 2)$

2. Write the equations for the following:

A. Vertical Stretch of 5; then a vertical translation of 2

B. Vertical Stretch of $\frac{1}{2}$; then vertical a translation of 1

C. Vertical translation of -3; then a vertical stretch of 2

D. Vertical translation of $\frac{1}{3}$; then a vertical stretch of 6

E. Vertical stretch of 1.97; then a vertical translation of 0.45

PARALLEL AND PERPENDICULAR LINES

A. PARALLEL LINES

Use Zap-a-graph to draw the following lines on your screen:

$y = 3x - 1$
$y = 3x + 1$
$y = 3x + 4$

1. What do the equations have in common?

____

2. What do the graphs have in common?

____

3. Why are these lines called **parallel**?

____

4. Give three equations of lines different from those above which are parallel.

________________, ________________, ________________
Graph them on the computer to check.

5. Write a statement to summarize what you learned.

____

____

B. PERPENDICULAR LINES

Use Zap-a-graph to graph the line $y = 2x + 1$
Pull down TRANSFORM and click on ROTATE 90°

*How are the two lines related?______________________________

*What is the equation of the new line? ________________

*What is the slope of the original line?________ the new line?_________

*Multiply the two slopes together. What is the result? ______________________

*Fill in the chart below:

Equation of Original Line	Slope 1	Equation of New Line after rotating	Slope 2	Product of Slopes Slope 1 X Slope 2
$y = x - 5$				
$y = -4x + 1$				
$y = \frac{1}{3}x - 2$				
$y = 0.4x - 1$				
$y = -0.75x + 2$				

*What do you notice about the product of the slopes?

We call these slopes **negative reciprocals** of each other.

Negative reciprocals are easier to recognize if we use change decimals to fraction form.

Fill in the chart below as a review of familiar decimal and fraction equivalents:

Fraction	Decimal	Fraction	Decimal	Fraction	Decimal
$\frac{1}{2}$		$\frac{2}{3}$		$\frac{1}{5}$	
$\frac{1}{3}$		$\frac{3}{4}$		$\frac{1}{8}$	
$\frac{1}{4}$		$\frac{3}{5}$		$\frac{3}{8}$	
$\frac{1}{6}$		$\frac{4}{5}$		$\frac{3}{7}$	
	$0.\overline{3}$		0.4		$0.\overline{6}$

C. Below are the equations of ten lines. Use the letters to indicate pairs of parallel and perpendicular lines.

a. $y = 3x - 7$

b. $y = 0.\overline{6}x$

c. $y = 4x + 7$

d. $y = -\frac{1}{3}x + 7$

e. $y = 0.\overline{6}x - 5$

f. $y = -3x + 7$

g. $y = 0.25x + 3$

h. $y = \frac{3}{2}x + 4$

Parallel lines: ______________________________

Perpendicular Line: ______________________________

HORIZONTAL AND VERTICAL LINES

A. GRAPHING HORIZONTAL LINES

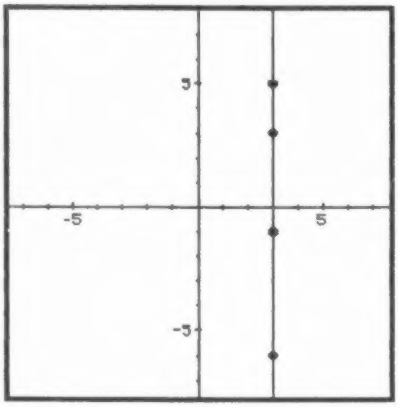
Use the ___LINE___ menu to graph the following:

$y = 0x - 3$
$y = 2 - 0x$
$y = 6$ (Graph as $y = 0x + 6$)

What is the term which describes these lines? ______________

What is the slope of these lines? _____

B. GRAPHING VERTICAL LINES



Name the points shown on the graph at the left:

(___,___) (___,___) (___,___) (___,___)

What do all the pairs have in common?

____

What is the slope of the line?____________

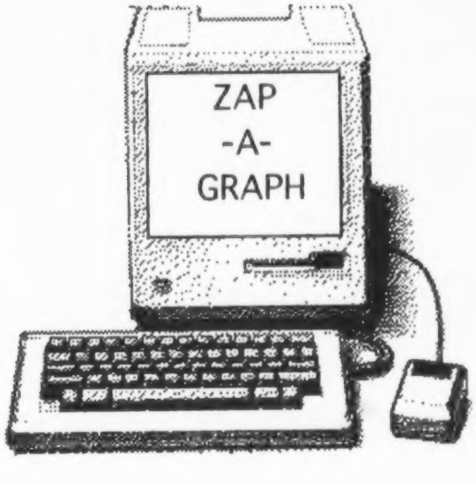
What is the equation of the line?_________

To graph the line, it is necessary to use the menu $Ax + By + C = 0$. Enter $A = 1$. $B = 0$, and $C = -3$. (Why?)

THE SIGNIFICANT SEVEN

RELATIONS AND FUNCTIONS

MATHEMATICS 20

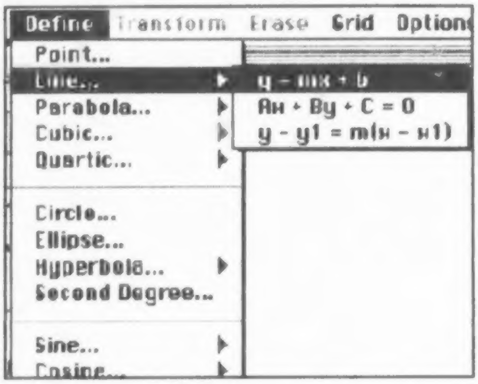


MARIAN OBERG
MAY, 1993

THE SIGNIFICANT SEVEN

A. GRAPHING THE BASIC FUNCTIONS WITH ZAP-A-GRAPH

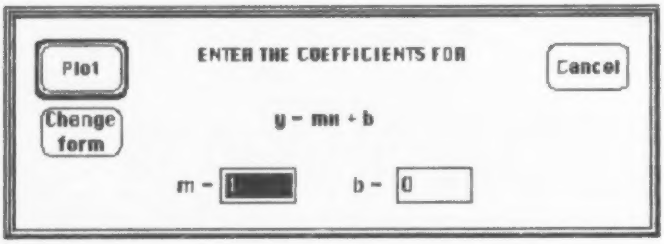
1. Line (y = x)


Define
Transform
Erase
Grid
Option
Point...
Line...
Parabola...
Cubic...
Quartic...
Circle...
Ellipse...
Hyperbola...
Second Degree...
Sine...
Cosine...
y = mx + b
Ax + By + C = 0
y - y1 = m(x - x1)

Pull down DEFINE

Click on LINE

Choose y = mx + b.

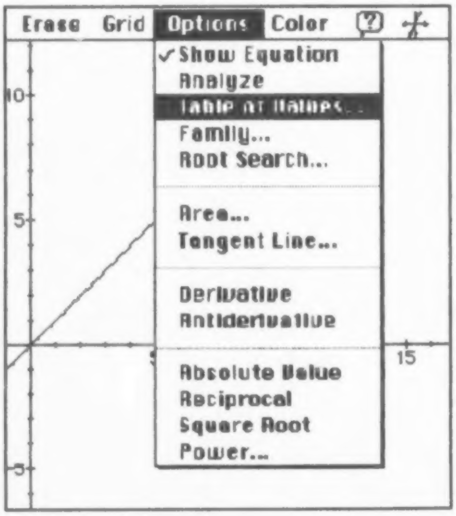

Plot
ENTER THE COEFFICIENTS FOR
Cancel
Change form
y = mx + b

m = 1 b = 0

Make sure m = 1 and
b = 0.

To change the numbers, use TAB to move between the values.

Press ENTER or RETURN when ready or click on PLOT.


Erase
Grid
Options
Color
✓Show Equation
Analyze
Table of Values...
Family...
Root Search...
Area...
Tangent Line...
Derivative
Antiderivative
Absolute Value
Reciprocal
Square Root
Power...
10
5
-5
15

Pull down the OPTIONS menu.

Click on TABLE OF VALUES

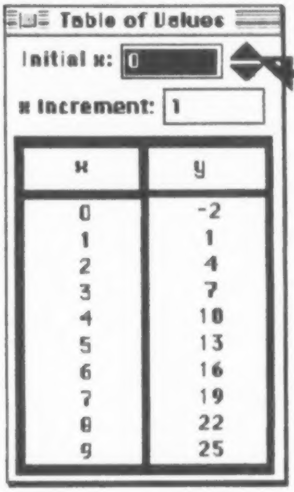


Table of Values

Initial x: 0

x Increment: 1

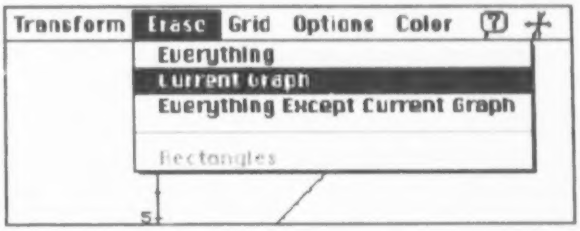
x	y
0	-2
1	1
2	4
3	7
4	10
5	13
6	16
7	19
8	22
9	25

Click on the arrows to change the values of x.

*Change the initial x to -5. What is the last x in the table? _____

Change the x-increment to 2. What range of values does y have?_______________

Pull down (ERASE) and choose (CURRENT GRAPH).


Transform
Erase
Grid
Options
Color
Everything
Current Graph
Everything Except Current Graph
Rectangles
5

???QUESTIONS???

1. The graph of a line can be expressed as $y = mx + b$. In this form, "m" represents _______________ and "b" represents __________________.

2. If a line has the equation $y = 2x - 3$, the slope is _____ and the y-intercept is _____.

3. You have graphed the line $y = x$ above. For this line, the slope is _____ and the y-intercept is _____.

4. There are two other menus to graph lines. How could the equation $y = x$ be rearranged to use the menu $Ax + By + C = 0$? _______________

5. To use the menu $(y - y_1) = m(x - x_1)$, a point on the graph is substituted for (x_1, y_1). What point could be used? _____

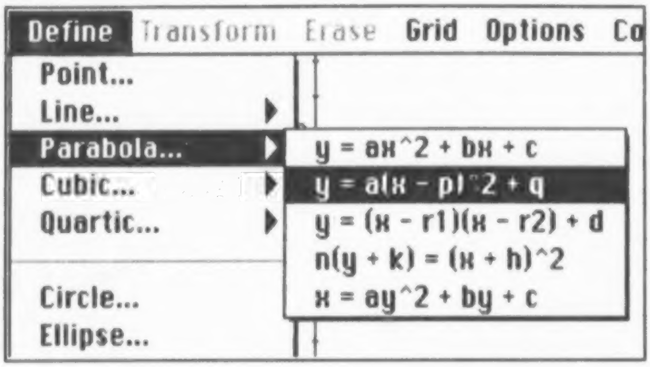
2. Parabola ($y = x^2$)

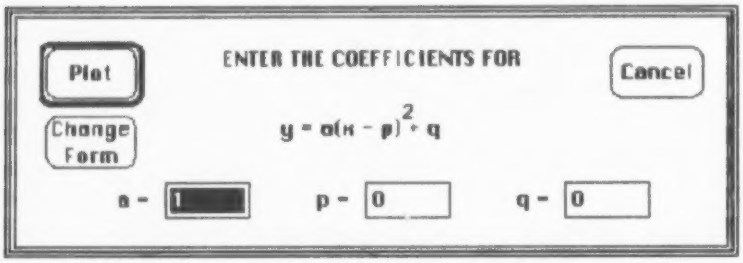
Pull down DEFINE

Click on PARABOLA .

Click on
$y = a(x-p)^2 + q$.

Make $a = 1$
$p = 0$
$q = 0$.


Define
Transform
Erase
Grid
Options
Co
Point...
Line...
Parabola...
Cubic...
Quartic...
Circle...
Ellipse...
y = ax^2 + bx + c
y = a(x - p)^2 + q
y = (x - r1)(x - r2) + d
n(y + k) = (x + h)^2
x = ay^2 + by + c


Plot
Change Form
Cancel

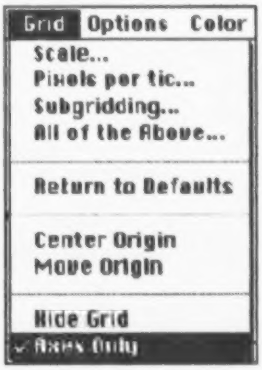
ENTER THE COEFFICIENTS FOR

$y = a(x - p)^2 + q$

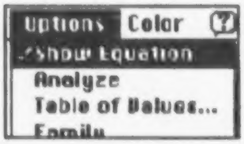
a = 1 p = 0 q = 0

Use TAB to move between values.

Click on PLOT or hit RETURN .


Grid
Options
Color
Scale...
Pixels per tic...
Subgridding...
All of the Above...
Return to Defaults
Center Origin
Move Origin
Hide Grid
✓Axes Only

By clicking on GRID and choosing AXES ONLY the check mark which normally appears can be turned off. When this happens, a grid appears. This can be useful for reading values from the graph.

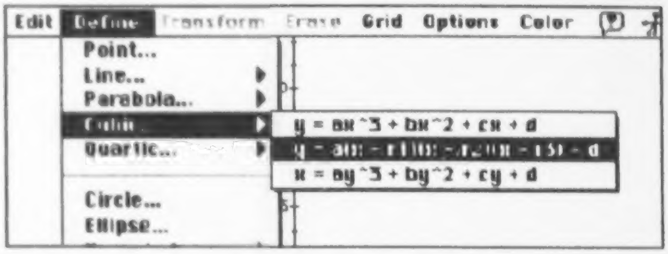

Options
Color
?
✓Show Equation
Analyze
Table of Values...
Family

The equation at the bottom of the screen is controlled by the SHOW EQUATION command listed under GRID . It can be turned on or off by clicking on it when highlighted.

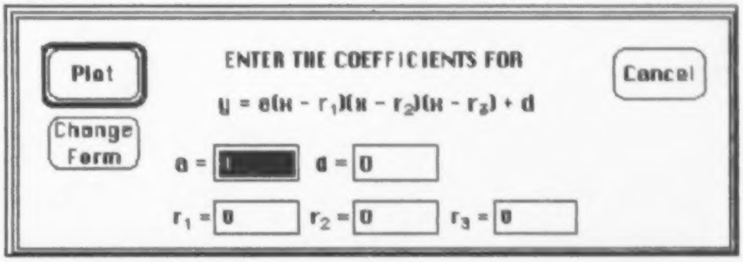
NOTICE: HOLDING DOWN ON THE MOUSE WILL GIVE YOU THE COORDINATES OF THE POINT AT WHICH THE ARROW CONTROLLED BY THE MOUSE IS POINTING.

3. Cubic ($y = x^3$)

Pull down DEFINE.
Choose CUBIC.


Edit
Define
Transform
Erase
Grid
Options
Color
Point...
Line...
Parabola...
Cubic...
Quartic...
Circle...
Ellipse...
y = ax^3 + bx^2 + cx + d
x = ay^3 + by^2 + cy + d

Choose the equation $y = a(x-r1)(x-r2)x-r3) + d$.
(which is really $y = a(x-r)^3 + d$ for us).


Plot
Change Form
Cancel

ENTER THE COEFFICIENTS FOR

$y = a(x - r_1)(x - r_2)(x - r_3) + d$

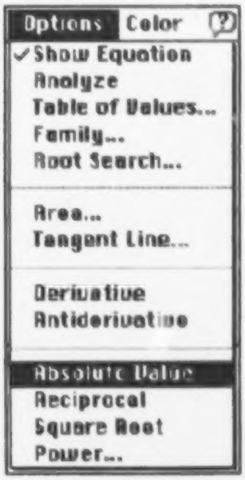
$a = 1$ $d = 0$

$r_1 = 0$ $r_2 = 0$ $r_3 = 0$

Enter $a = 1$
$r1 = 0$
$r2 = 0$
$r3 = 0$
$d = 0$
as shown at the left.

PLOT.

4. Absolute Value ($y = |x|$)


Options
Color
✓Show Equation
Analyze
Table of Values...
Family...
Root Search...
Area...
Tangent Line...
Derivative
Antiderivative
Absolute Value
Reciprocal
Square Root
Power...

FOLLOW THESE STEPS TO GRAPH ABSOLUTE VALUE:

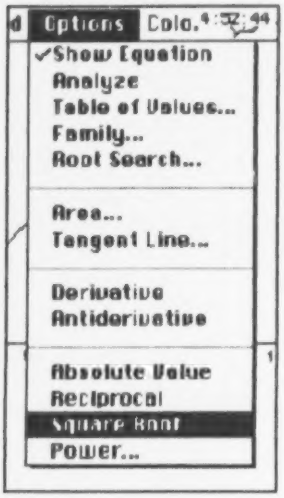
1. Graph the equation of $y = x$. (See #1 if you've forgotten how.)

2. Pull down the OPTIONS menu.
   Choose ABSOLUTE VALUE.

3. Pull down ERASE.
   Choose EVERYTHING EXCEPT CURRENT GRAPH.

This should leave the absolute value [V] graph remaining on the screen.

5. Square Root ($y = \sqrt{x}$)


Options
Colo.
✓Show Equation
Analyze
Table of Values...
Family...
Root Search...
Area...
Tangent Line...
Derivative
Antiderivative
Absolute Value
Reciprocal
Square Root
Power...

FOLLOW THESE STEPS TO GRAPH SQUARE ROOT:

1. Graph y = x.

2. Pull down the OPTIONS menu.
   Click on SQUARE ROOT.

3. Use the ERASE menu to remove EVERYTHING EXCEPT THE CURRENT GRAPH.

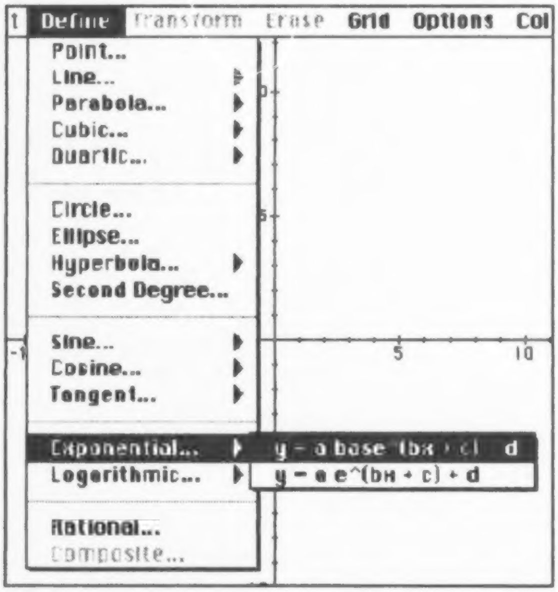
This should leave the square root graph remaining on the screen.

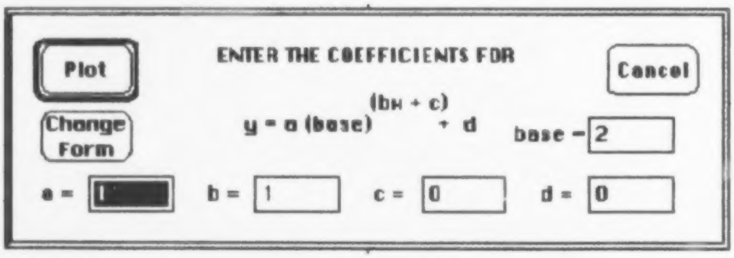
6. Exponential Graph ($y = 2^x$)

Pull down DEFINE

Choose EXPONENTIAL.

Click on the first option.


Define
Transform
Erase
Grid
Options
Col
Point...
Line...
Parabola...
Cubic...
Quartic...
Circle...
Ellipse...
Hyperbola...
Second Degree...
Sine...
Cosine...
Tangent...
Exponential...
Logarithmic...
Rational...
Composite...
y = a base^(bx + c) + d
y = a e^(bx + c) + d
5
10


Plot
Change Form
Cancel

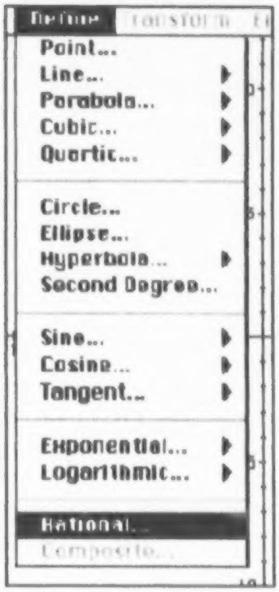
ENTER THE COEFFICIENTS FOR

$y = a\,(\text{base})^{(bx + c)} + d$ base = 2

a = 1 b = 1 c = 0 d = 0

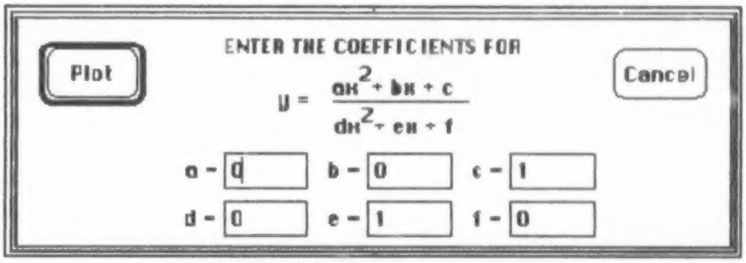
Choose a = 1
b = 1
c = 0
d = 0
base = 2

7. Reciprocal $(y = \frac{1}{x})$


Define
Transform
Point...
Line...
Parabola...
Cubic...
Quartic...
Circle...
Ellipse...
Hyperbola...
Second Degree...
Sine...
Cosine...
Tangent...
Exponential...
Logarithmic...
Rational...
Composite...

Pull down DEFINE.

Choose RATIONAL.


Plot
Cancel

ENTER THE COEFFICIENTS FOR

$$y = \frac{ax^2 + bx + c}{dx^2 + ex + f}$$

a = 0 b = 0 c = 1

d = 0 e = 1 f = 0

Choose the values

a = 0 b = 0

c = 1 d = 0

e = 1 f = 0

PLOT.

???QUESTIONS???

1. This graph is composed of two parts and is sometimes called a rectangular hyperbola.

   a. There is one value of x which is excluded from the domain. What is it? Why is impossible to define the function for that value?

   b. There is one value of y which is excluded from the range? What is it? Why is it excluded?

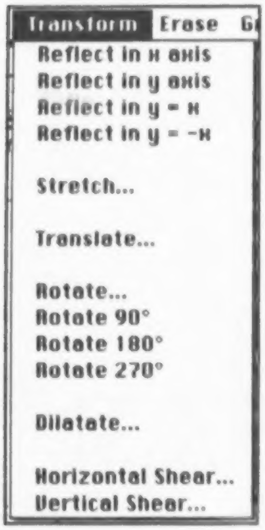
   Sometimes lines, called asymptotes, are drawn as guidelines using the values in "a" and "b", demonstrating that the function approaches these values as we move away from the centre of the system.
   What are the asymptotes for this graph?

B. TRANSLATING GRAPHS WITH ZAP-A-GRAPH

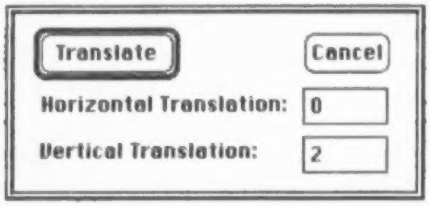
1. Vertical and Horizontal Translations

You have already worked with the line and the parabola extensively, so we'll start with a different graph. Graph $y = \sqrt{x}$ on Zap-a-graph.

What point can be used as a reference point to follow the movement of the graph?__________


Transform
Erase
Gi
Reflect in x axis
Reflect in y axis
Reflect in y = x
Reflect in y = -x
Stretch...
Translate...
Rotate...
Rotate 90°
Rotate 180°
Rotate 270°
Dilatate...
Horizontal Shear...
Vertical Shear...

Pull down (TRANSFORM) and click on (TRANSLATE).


Translate
Cancel

Horizontal Translation: 0

Vertical Translation: 2

(TAB) to Vertical Translation.

Enter 2 (the first vertical translation given in the chart below) and record the results. Use the reference point to help you track the movement of the graph.

Zap-a-graph always acts on the most recent graph it has drawn so (ERASE) (CURRENT GRAPH) after each entry to see the effect on $y = \sqrt{x}$.

Vertical Translation	New location of reference point	New Equation	New position of graph after translation
2			
-3			
4			
-1			

*Discuss what appears to have happened to both the graph and the equation when these translations were carried out. ______________________________

____

Carry out the translations indicated in the chart and fill in the blanks:

Starting Equation	Apply a Vertical Translation of	New Equation	Sketch of Graph
$y = x$	3		
$y = \lvert x \rvert$	-2		
$y = \frac{1}{x}$	5		

???QUESTIONS???

1. Describe what happened to the equation and the graph when the graph was translated vertically.______________________________

   ____

2. If the vertical translation is "a", then the new function is y = ________

3. If the original function is called $f(x)$, which of the following describes the new function?
   $y = f(x + a)$ $y = f(x) + a$ $y = f(x) - a$ $y = af(x)$

NEXT

Follow the steps given above, but this time, try horizontal translations on $y = \sqrt{x}$.
Fill in the chart below:

Horizontal Translation	New Position of Reference Point	New Equation	New Position of Graph after Translation
2			
-1			
6			
-3			

???QUESTIONS???

1. Try out horizontal translations on at least one other relation. Describe the relations you used, the amount of translations, and the results. Sketch.

2. Summarize what happened to the graphs and the equations when the graphs were translated horizontally.

____

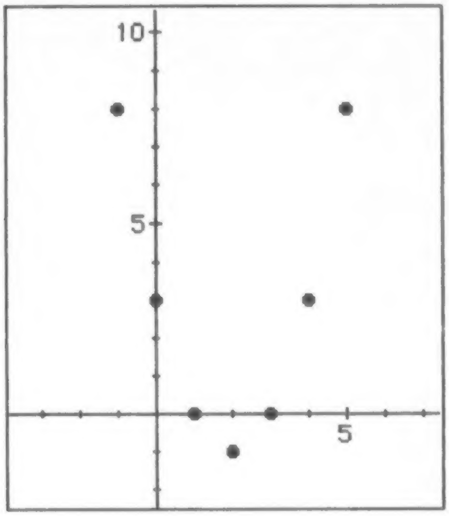
____

3. If the horizontal translation was "a", the new function is y = ________ .

4. If the original function is represented by f(x), which of the following describes the new function:
$y = f(x + a)$ $y = f(x) + a$ $y = f(x - a)$ $y = af(x)$

GUESS MY EQUATION!

1.



2.

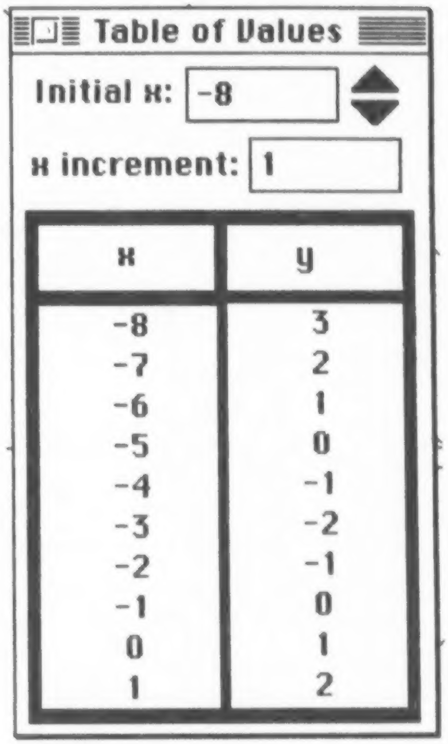


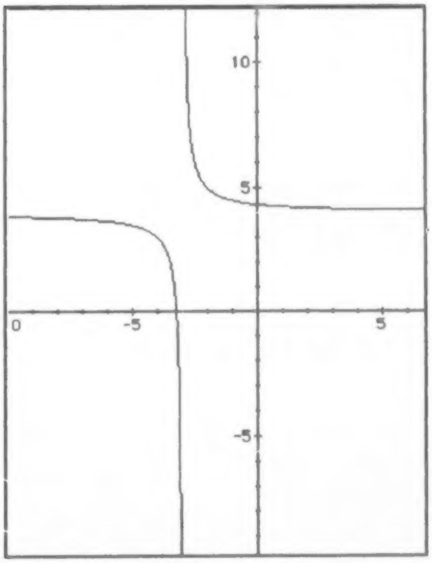
Table of Values

Initial x: -8

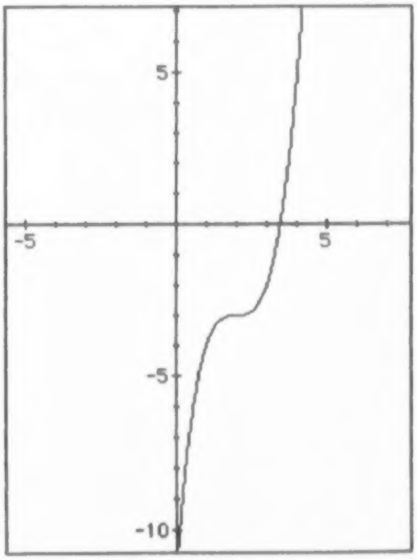
x increment: 1

x	y
-8	3
-7	2
-6	1
-5	0
-4	-1
-3	-2
-2	-1
-1	0
0	1
1	2

3.

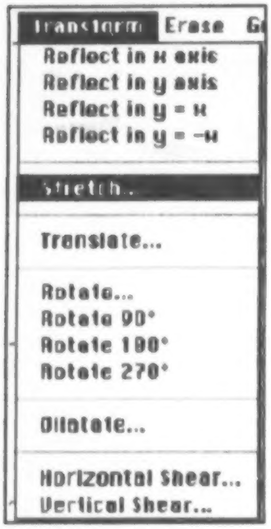


4.



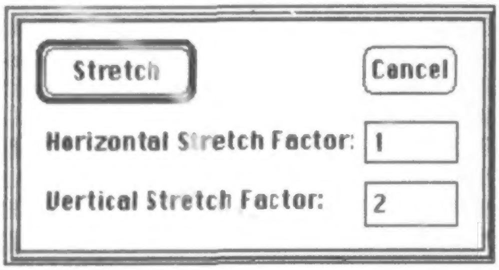
2. Vertical Stretch

Begin with the graph of y = |x| .


Transform
Erase
Reflect in x axis
Reflect in y axis
Reflect in y = x
Reflect in y = -x
Stretch...
Translate...
Rotate...
Rotate 90°
Rotate 180°
Rotate 270°
Dilatate...
Horizontal Shear...
Vertical Shear...

Pull down TRANSFORM and click on STRETCH .

Enter a Vertical Stretch Factor of 2.

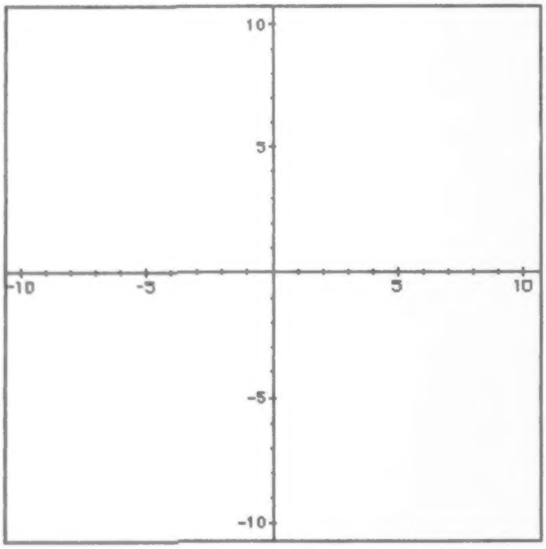

Stretch
Cancel

Horizontal Stretch Factor: 1

Vertical Stretch Factor: 2

???QUESTIONS??? Sketch all examples on the grid below. LABEL.

1. What is the result of the vertical stretch of 2? (to the graph and the equation)



2. Turn on the Table of Values. Begin at the reference point. Compare the changes in y as x changes by 1 to the changes in your graph in part A.

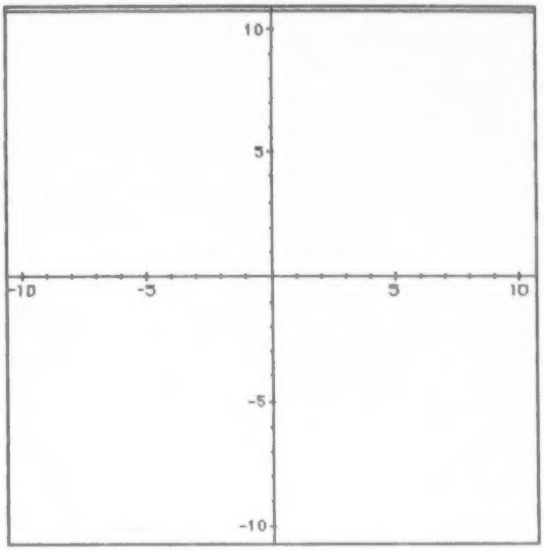
ERASE CURRENT GRAPH .

3. Enter a vertical stretch factor of 5. Sketch the result on the graph above. What is the result? Check the Table of Values. How has it changed? ERASE CURRENT GRAPH .

4. What happens if the vertical stretch factor is -3? Sketch and label the graph with its equation.

5. What happens if the vertical stretch factor is $\frac{1}{2}$, or $-\frac{1}{2}$? Sketch.

6. If f(x) is stretched vertically by "a", what is the new function?

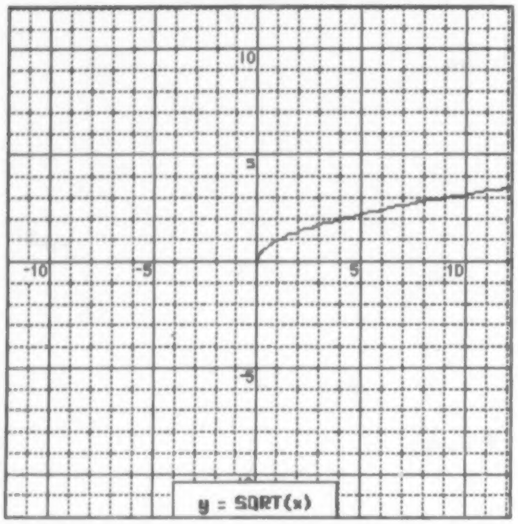


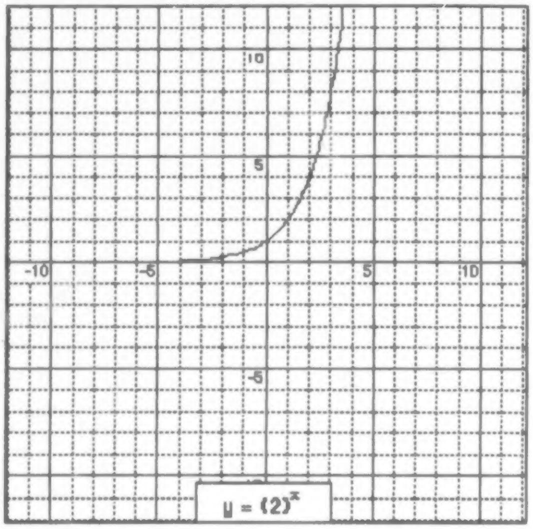
ASSIGNMENT

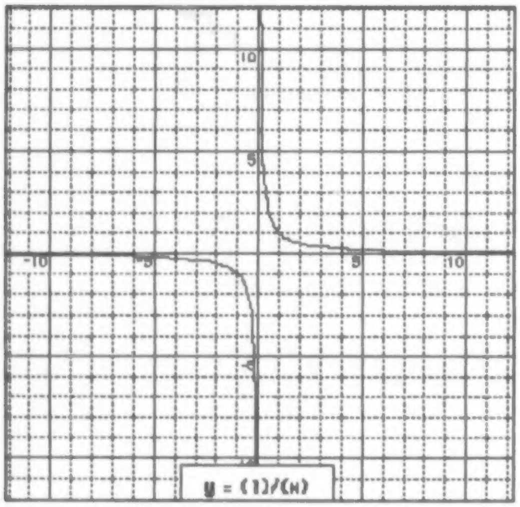
Apply vertical stretches of 2, $-\frac{1}{3}$, and -1 to each of the functions shown on the next page. Sketch the result on the same grid as the original.

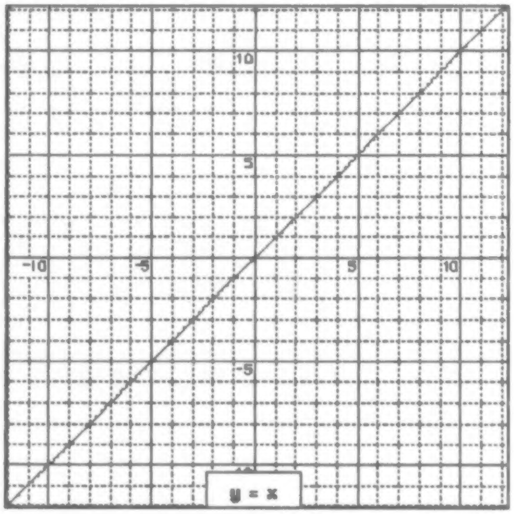
(ERASE) (CURRENT GRAPH) between examples when using the computer.

SKETCH CAREFULLY, MAKING SURE THE VERTICAL CHANGE IS MARKED ACCURATELY.

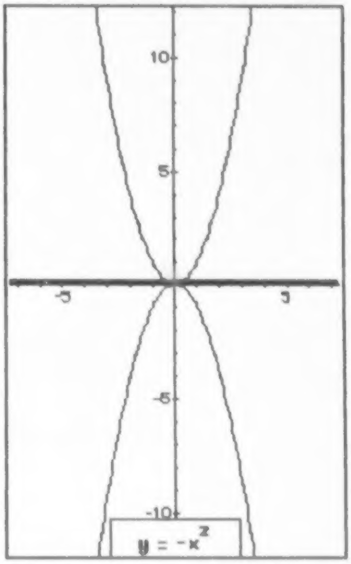

10
5
-10
-5
5
10
-5
y = SQRT(x)


10
5
-10
-5
5
10
-5
y = (2)^x


10
5
-10
-5
5
10
-5
y = (1)/(x)


10
5
-10
-5
5
10
-5
y = x

C. Reflections Using Zap-a-graph

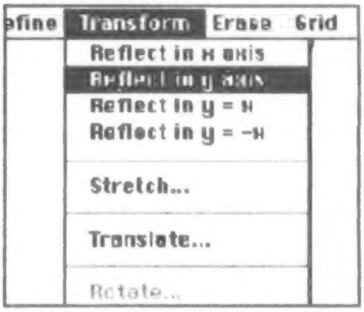


When we used a vertical stretch of (-1) the graph appeared to be reflected in the x-axis as shown to the right.

The equation, given by $y = af(x)$, becomes $y = (-1)f(x)$ or $-f(x)$.

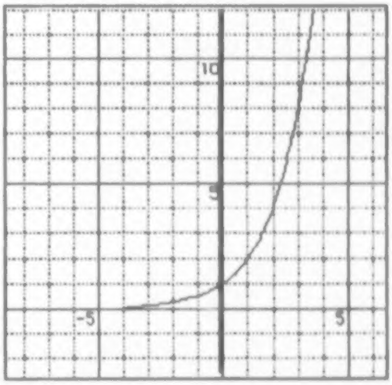
What happens when we reflect the function in the y-axis?

To find out, we need to use a function which is not symmetric about the y-axis, or it will be impossible to tell what has happened.


efine
Transform
Erase
Grid
Reflect in x axis
Reflect in y axis
Reflect in y = x
Reflect in y = -x
Stretch...
Translate...
Rotate...

Graph $y = 2^x$.

Pull down the TRANSFORM menu and click on REFLECT IN Y AXIS.



Sketch the result on the graph to the right. What is the new equation?

Try a similar reflection with $y = \sqrt{x}$.

If the original function is $y = f(x)$, then the new function will be: (choose one)

$y = -f(x)$ $y = f(x - 1)$ $y = f(-x)$

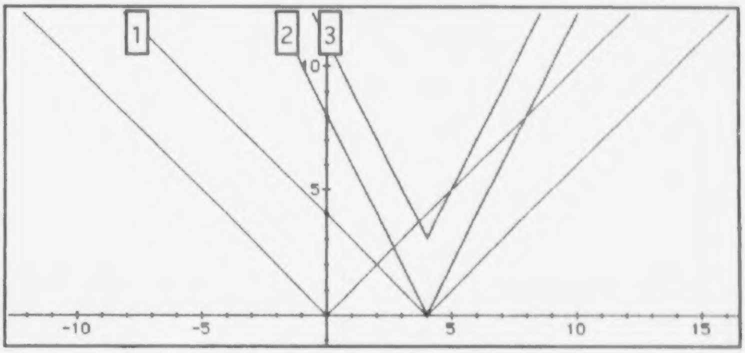
D. Multiple Transformations with Zap-a-graph

Although many of the functions may be graphed directly, using the menus given with Zap-a-graph, some cannot. If we wished to graph the function ($y = 2|x - 4| + 3$) we must do it using translations.

The order in which we carry out the translations makes a difference. Try this order:

1. horizontal translation
or2. vertical stretch
3. vertical translation

You will notice that doing the vertical translation earlier will result in a different graph.



To sketch translations without Zap-a-graph, try the following steps:

1. Sketch the basic function.
2. Sketch the stretched function before translation.
3. Apply the horizontal and vertical translations.

Example: $y = \frac{1}{2}(x + 2)^3 - 4$

1. Sketch x^3.

2. Sketch $\frac{1}{2}x^3$

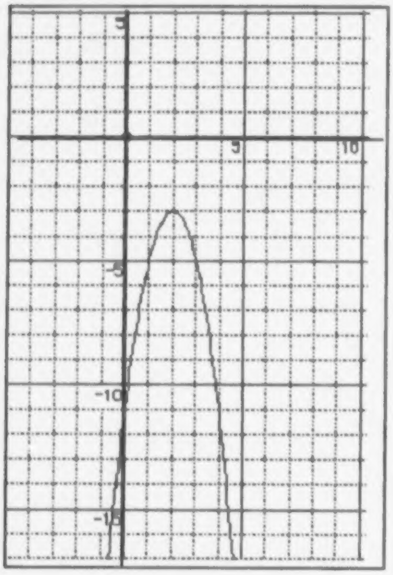
3. Plot the new reference point at (-2, -4).

4. Redraw the shape from step #2 relative to the new reference point

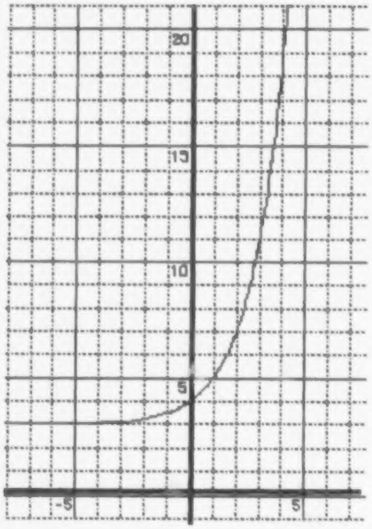
Guess My Equation

Determine the equation of the following from the graph shown:

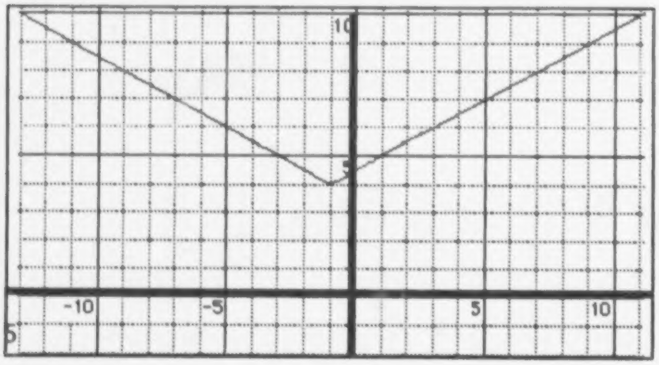
1.



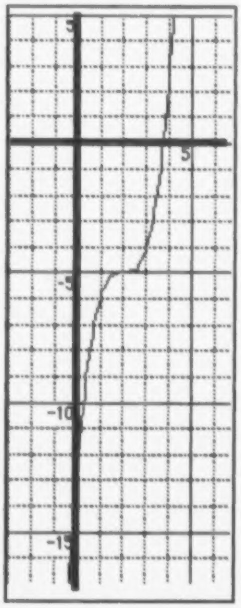
2.



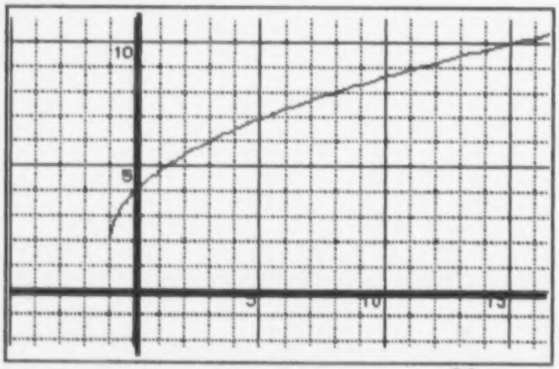
3.



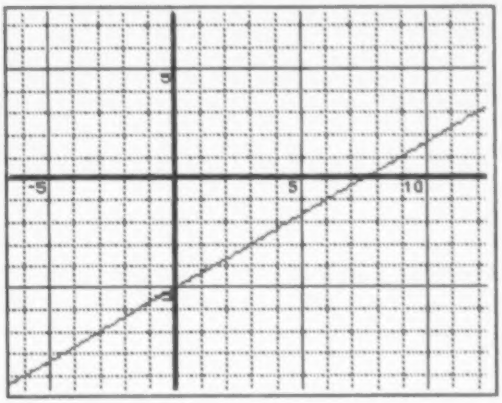
4.



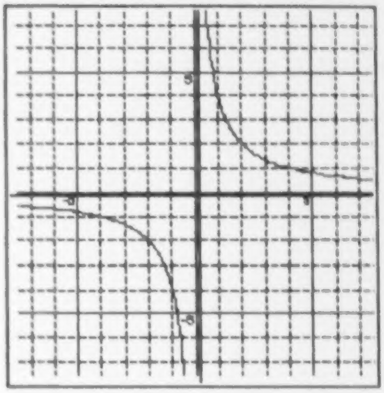
5.



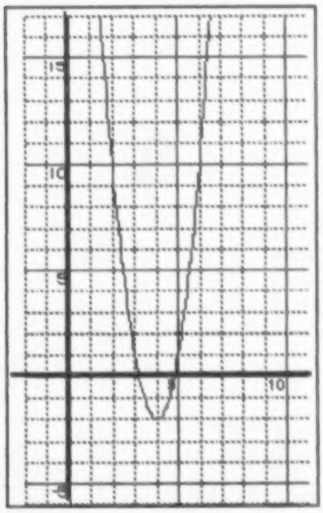
6.



7.



8.



9.

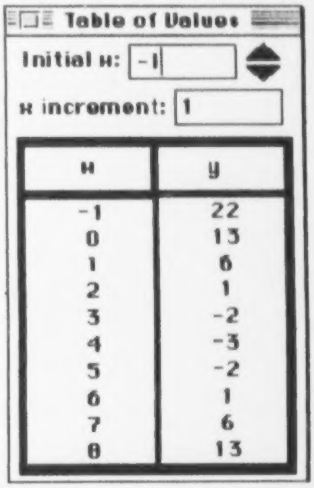


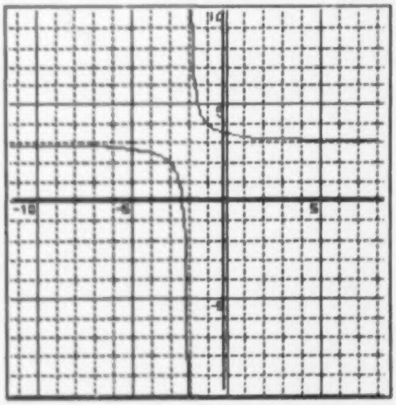
Table of Values

Initial x: -1

x increment: 1

x	y
-1	22
0	13
1	6
2	1
3	-2
4	-3
5	-2
6	1
7	6
8	13

10.

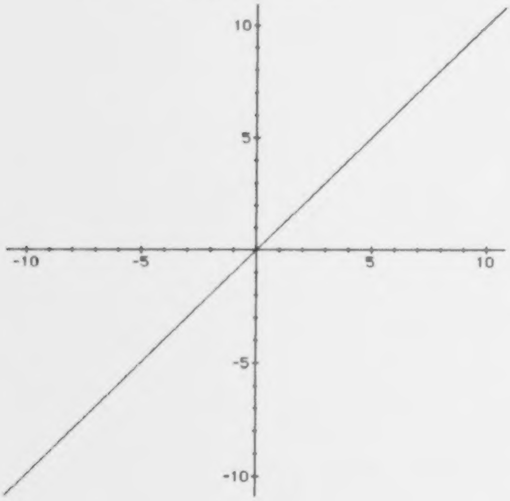


E. Inverses of Functions and Relations with Zap-a-graph

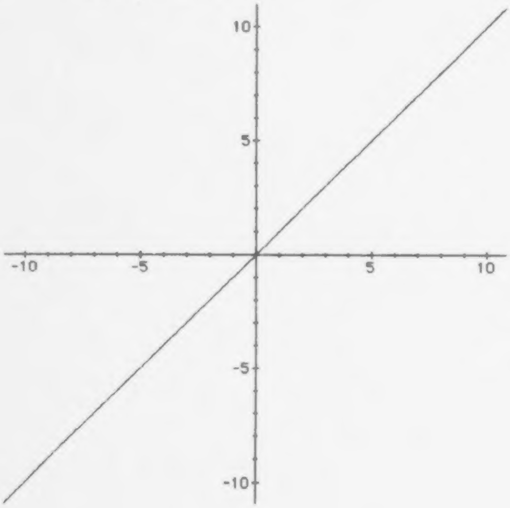
Zap-a-graph will allow you to reflect a graph drawn from the DEFINE menu in the line $y = x$, thus producing the inverse. The equation will be shown as well.

Enter the following equations and reflect them in $y = x$. Sketch the original and the inverse; label both with their equations. The line $y = x$ has been drawn on each graph.

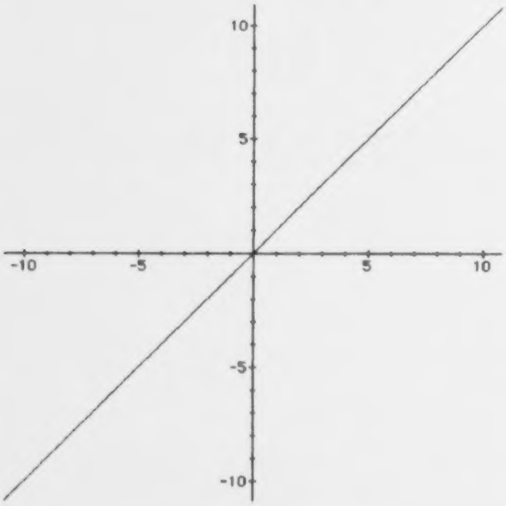
1. $y = 3x^2 - x + 5$



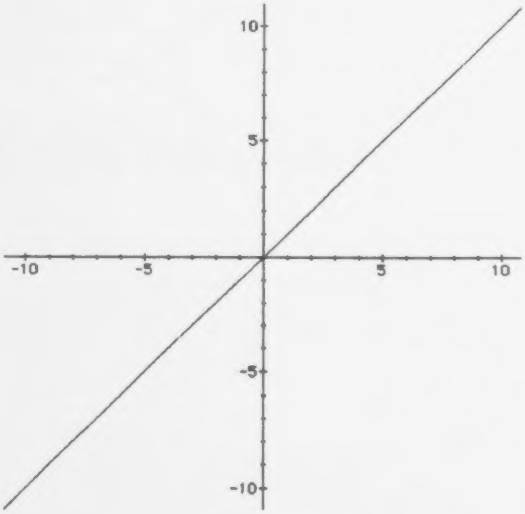
2. $4x - y - 3 = 0$



3. $y = \frac{(2x - 6)}{(x - 4)}$



4. $y = 2x^3 - 3x^2 + x - 6$



EVALUATION FORM FOR STUDENTS LEARNING ABOUT LINES

Thank you for participating in this project and for taking a few minutes to complete the following questions. I will be using your answers to help me improve the worksheets and as part of the information for my thesis.

PLEASE DO NOT PUT YOUR NAME ON THIS SHEET. SIGN THE SEPARATE FORM BESIDE THE NUMBER OF THIS SHEET. BY SIGNING, YOU ARE AGREEING TO LET ME USE YOUR ANSWERS (YOUR NAME WILL NOT BE USED) IN ANY REPORTS OF THIS RESEARCH.

1. Age (as of June 30, 1993)______ Gender (Female, Male)

2. Have you used computers in any mathematics class before? When and what for?

3. Do you use computers often in other school classes during this school year? For what purpose?

4. Do you use computers often outside of school? For what purpose?

5. Please rate the exercises as satisfactory(S) or unsatisfactory(U) on the following criteria: (Feel free to comment further below.)

_____A. The material was easy to read.

_____B. The procedures were explained well enough so that I could work on my own.

_____C. There were enough examples so I could come to conclusions.

_____D. The computer facilities allowed me to work well.

_____E. I worked (with a partner/alone/as part of a larger group) (*Underline one.*) This was the best way to work.
COMMENT BELOW IF YOU WISH.

6. Did you explore any parts of the program which were not part of the lesson?

7. What was the best part of using the computer to study coordinate geometry?

8. What was the worst part of using the computer to study coordinate geometry?

9. If you were the teacher planning this unit, which method do you think might be the most meaningful for students?

    A. computer with worksheets; individual help from teacher

    B. teacher explanation with computer demonstration

    C. teacher explanation with overhead or board demonstration

    D. group work with graphing on paper

    E. graphing calculators with teacher instruction

    F. graphing calculators with worksheets; individual teacher help

    G. other (Please explain.)

10. Please add any further remarks which you think might be useful.

Focus Questions for Teachers

1. Demographic Information (education, teaching experience, gender).

2. What is the best feature about teaching this unit with computers?

3. What is the worst feature of this method?

4. If you were to rewrite this unit, what would you change?

5. Do you think that other teachers would be comfortable with this method? What advice would you give them?

6. In your opinion, how will the mathematics curriculum be affected by the use of technology in the classroom?

7. Do you feel that the use of technology will change our expectations of students? For example, will we expect a higher level of cognitive activity?

8. In the past what forms of inservice or professional development activities have you found most helpful?

   a. one-day seminars
   b. conferences (which ones?)
   c. reading (what materials?)
   d. discussion with colleagues?
   e. credit/non-credit courses (which ones?)

9. What events/activities encourage mathematics teachers to try new ideas?

10. What keeps mathematics teachers from trying new ideas in the classroom?

11. If you were asked to provide professional development for mathematics teachers interested in using graphing software, what features would you try to include?

12. Do you see many examples of collegiality among your colleagues? How can this be encouraged among teachers without mandating it?

Coordinate Geometry and Graphing
Program Emphasis - 20%

Concept	Skills	Problem Solving/ Technology	TRM Page
Students will be expected to demonstrate an understanding that points and lines can be represented on a Cartesian plane and that they have characteristics which can be defined and measured.	Students will be expected to use the formula for the distance between two points.	Students will be expected to deduce the distance formula from the Pythagorean theorem. Students will be expected to solve problems that involve the use of the distance formula.	pp. 76–77
	Students will be expected to use the formula for the coordinates of the midpoint of a line segment.	Students will be expected to verify the midpoint formula. Students will be expected to solve problems that involve the use of the midpoint formula.	pp. 78–79
	Students will be expected to use the formula for the slope of a line passing through two points.	Students will be expected to solve problems that involve the use of the formula for the slope of the segment between two points. Students will be expected to solve problems that involve the relationship between the slopes of parallel and perpendicular lines.	pp. 80–83
	Students will be expected to determine whether points in a plane are collinear.	Students will be expected to solve problems that involve the necessary and sufficient conditions for collinearity.	p. 84
Students will be expected to demonstrate an understanding that the ordered pairs which satisfy a linear equation, $Ax + By + C = 0$, correspond to coordinates on the Cartesian plane and can be used to graph the equation on the Cartesian plane.	Students will be expected to find sets of ordered pairs that satisfy linear equations.		pp. 85–86
	Students will be expected to graph linear equations by plotting sets of ordered pairs that satisfy the equation.		
	Students will be expected to find the x- and y- intercepts of linear equations.		p. 87
	Students will be expected to graph linear equations by use of the x- and y- intercepts.		
	Students will be expected to write linear equations in the slope-intercept form, $y = mx + b$.		pp. 88–90
	Students will be expected to graph linear equations using the slope and the y-intercept or another point that satisfies the equation.	Students will be expected to design and carry out an investigation involving the use of scientific calculators or computers to determine the effects of the parameters m and b on the graphs of linear equations.	
	Students will be expected to identify and graph equations of lines parallel to the x- and y-axes.		p.91
Students will be expected to demonstrate an understanding that lines in a plane can be uniquely determined.	Students will be expected to graph and write the equation of a line given the conditions which uniquely determine it.	Students will be expected to solve problems that involve conditions which uniquely determine a line.	pp. 92–93

FUNCTIONS AND RELATIONS PROGRAM EMPHASIS - 16%

Concept	Skills	Problem Solving/ Technology
Students will be expected to demonstrate an understanding that certain observed real-world phenomena are quantitatively related to each other and that theses relations can be described graphically, with sets of ordered pairs, rules and equations.	Students will be expected to graph relations that describe physical phenomena or everyday situations. Students will be expected to determine the domain and the range of relations algebraically and from given graphs.	Students will be expected to solve problems by graphing and interpreting the graphs that describe physical phenomena and everyday occurrences
Students will be expected to demonstrate an understanding that for some relations, called functions, the value of the independent variable (domain) uniquely determines the value of the function (range, dependent variable).	Students will be expected to represent simple mathematical situations such as direct, inverse and partial variations with tables of values, identify the dependent and independent variables, and express the domain ad range, appropriately noting any restrictions.	
	Students will be expected to interpolate and extrapolate from the graphs of functional relationships.	
	Students will be expected to use functional notation and graphs to describe functional relationships.	Students will be expected to solve problems algebraically or by the use and interpretation of graphs that represent functions.
	Students will be expected to determine those relations that are functions.	Students will be expected to develop and explain tests which could be used to determine if any relation is or is not a function.
	Students will be expected to illustrate and recognize different kinds of functions algebraically and graphically from the following list: linear functions (including identity and constant functions), polynomial functions (including quadratic and cubic functions), reciprocal functions, absolute value functions, and exponential functions.	Students will be expected to draw and analyze the graphs of functions using calculators or computers.

FUNCTIONS AND RELATIONS (CONT'D)
PROGRAM EMPHASIS - 16%

Concept	Skills	Problem Solving/ Technology
Students will be expected to demonstrate an understanding that for some relations called functions, the value of the independent variable (domain) uniquely determines the value of the function (range, dependent variable).	Students will be expected to identify the zeros of a function as the x-intercepts of its graph. Students will be expected to write, and sketch the graphs of, the inverses of relations and functions.	Students will be expected to solve find the zeros of a function by analyzing its graph and its value for various replacements of the independent variable, using calculators or computers.
Students will be expected to demonstrate an understanding of how particular parameters can be used to effect translations, reflections or vertical stretching of the graph of any function.	Students will be expected to describe the transformation effects on the graph of $y = f(x)$ of the parameters a and b in $y = f(x-a) + b$. Students will be expected to describe the transformation effect on the graph of $y = f(x)$ of the parameter c in $y = cf(x)$. Students will be expected to describe and sketch the graphs of $y = cf(x-a) + b$ by applying the transformation effects of a, b and c on the graph of $y = f(x)$.	Students will be expected to perform an investigation to determine the effects of the parameters a and b on the graph of $y = f(x-a) + b$. Students will be expected to perform an investigation to determine the effect of the parameter c on the graph of $y = cf(x)$. Students will be expected to predict the graphs of functions written in the form $y = cf(x-a) + b$ given the graph of $y = f(x)$, and verify using calculators or computers.